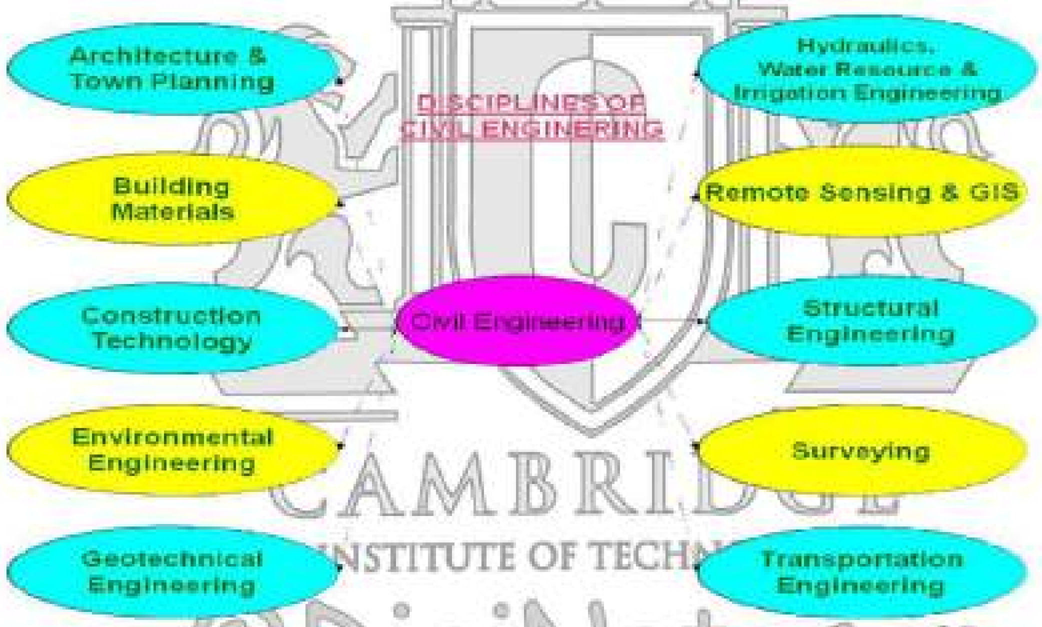
MODULE -1

Introduction to Civil Engineering &Engineering Mechanics

1 Introduction

Civil engineers have one of the world's most important jobs: they build our quality of life. With creativity and technical skill, civil engineers plan, design, construct and operate the facilities essential to modern life, ranging from bridges and highway systems to water treatment plants and energy efficient buildings. Civil engineers are problem solvers, meeting the challenges of pollution, traffic congestion, drinking water and energy needs, urban development and community planning. Civil engineering is an umbrella field comprised of many related specialties. The following figure shows the broad categories of fields under civil engineering.



An Engineer is a person who plays a key role in such activities.

1.1.1 Civil Engineering: It is the oldest branch of professional engineering, where the civil engineers are concerned with projects for the public or civilians.

The role of civil engineers is seen in every walk of life or infrastructure development activity such as follows:-

1. Providing shelter to people in the form of low cost houses to high rise apartments.
2. Laying ordinary village roads to express highways.
3. Constructing irrigation tanks, multipurpose dams & canals for supplying water to agricultural fields.
4. Supplying safe and potable water for public & industrial uses.

5. Protecting our environment by adopting sewage treatment & solid waste disposal techniques.
6. Constructing hydro-electric & thermal-power plants for generating electricity.
7. Providing other means of transportation such as railways, harbour & airports.
8. Constructing bridges across streams, rivers and also across seas.
9. Tunneling across mountains & also under water to connect places easily & reduce distance.

As seen above, civil engineering is a very broad discipline that incorporates many activates in various fields. However, civil engineers specialize themselves in one field of civil engineering. The different fields of civil engineering and the scope of each can be briefly discussed as follows.

1. **Surveying**: It is a science and art of determining the relative position of points on the earth's surface by measuring distances, directions and vertical heights directly or indirectly. Surveying helps in preparing maps and plans, which help in project implementation. (setting out the alignment for a road or railway track or canal, deciding the location for a dam or airport or harbour) The cost of the project can also be estimated before implementing the project. Now-a-days, using data from remote sensing satellites is helping to prepare maps & plans & thus cut down the cost of surveying.

2. **Geo-Technical Engineering (Soil Mechanics)**: Any building, bridge, dam, retaining wall etc consist of components like foundations. The foundation is laid from a certain depth below the ground surface till a hard layer is reached. The soil should be thoroughly checked for its suitability for construction purposes. The study dealing with the properties & behaviour of soil under loads & changes in environmental conditions is called geo-technical engineering. The knowledge of the geology of an area is also very much necessary.

3. **Structural Engineering**: A building or a bridge or a dam consists of various elements like foundations, columns, beams, slabs etc. These components are always subjected to forces. It becomes important to determine the magnitude & direction the nature of the forces and acting all the time. Depending upon the materials available or that can be used for construction, the components or the parts of the building should be safely & economically designed. A structured engineer is involved in such designing activity. The use of computers in designing the members, is reducing the time and also to maintain accuracy.

4. **Transportation Engineering**: The transport system includes roadways, railways, air & waterways. Here the role of civil engineers is to construct facilities related to each one. Sometimes crucial sections of railways & roads should be improved. Roads to remote places should be developed. Ports & harbors should be designed to accommodate, all sizes of vehicles. For an airport, the runway & other facilities such as taxiways, terminal buildings, control towers etc. should be properly designed.

5. **Irrigation & Water resources engineering (Hydraulics Engineering)**: Irrigation is the process of supplying water by artificial means to agricultural fields for raising crops. Since rainfall in an area is insufficient or unpredictable in an area, water flowing in a river can be stored by constructing dams and diverting the water into the canals & conveyed to the agricultural fields. Apart from dams & canals other associated structures

like canals regulators, aqua ducts, weirs, barrages etc. are also necessary. Hydro electric power generation facilities are also included under this aspect.

6. **Water Supply and Sanitary Engineering (Environmental Engineering)**: People in every village, town & city need potable water. The water available (surface water & ground water) may not be fit for direct consumption. In such cases, the water should be purified and then supplied to the public. For water purification, sedimentation tanks, filter beds, etc. should be designed. If the treatment plants are for away from the town or city, suitable pipelines for conveying water & distributing it should also be designed. In a town or city, a part of the water supplied returns as sewage. This sewage should be systematically collected and then disposed into the natural environment after providing suitable treatment. The solid waste that is generated in a town or locality should be systematically collected and disposed off suitably. Before disposal, segregation of materials should be done so that any material can be recycled & we can conserve our natural resources.
7. **Building Materials & Construction Technology:** Any engineering structure requires a wide range of materials known as building materials. The choice of the materials is wide & open. It becomes important for any construction engineer to be well versed with the properties & applications of the different materials. Any construction project involves many activities and also required many materials, manpower, machinery & money. The different activities should be planned properly; the manpower, materials & machinery should be optimally utilized, so that the construction is completed in time and in an economical manner. In case of large construction projects management techniques of preparing bar charts & network diagrams, help in completing the project orderly in time.

1.1.2 Effects of Infrastructure development on the Socio-economic development of a country:

The term infrastructure is widely used to denote the facilities available for the socio-economic development of a region. The infrastructure facilities to be provided for the public include:

1. Transport facilities
2. Drinking water and sanitation facilities
3. Irrigation facilities
4. Power generation & transmission facilities
5. Education facilities
6. Health care facilities
7. Housing facilities
8. Recreation facilities

The well being of a nation is dependent on the quality & the quantity of the above services that are provided to the public. Development of infrastructure has number of good effects which can be listed as follows.

1. It is a basic necessity for any country or state.
2. It forms a part of business, research & education.
3. It improved health care & Cultural activities.

4. It provided housing & means of communication to people.
5. It provided direct employment to many number of skilled, semiskilled & unskilled laborers.
6. It leads to the growth of associated industries like cement, steel, glass, timber, plastics, paints, electrical goods etc.
7. It helps in increasing food production & protection from famine.
8. Exporting agricultural goods can fetch foreign currency.

Some ill effects of infrastructure development can also be listed as follows:

1. Exploitation of natural resources can lead to environmental disasters.
2. Migration of people from villages to towns & cities in search of job takes place.
3. Slums are created in cities.
4. It becomes a huge financial burden on the government and tax payers.

ENGINEERING MECHANICS

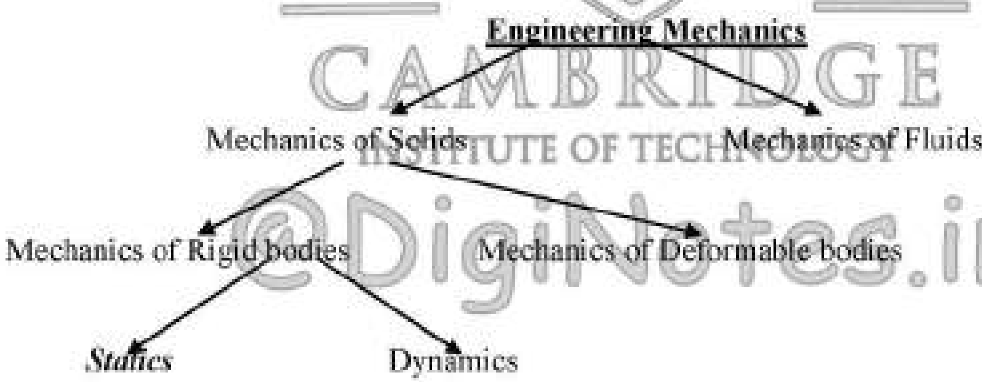
MECHANICS

It's a branch of science, which deals with the action of forces on bodies at rest or in motion.

ENGINEERING MECHANICS

It deals with the principles of mechanics as applied to the problems in engineering

Engineering Mechanics deals with the application of principles of mechanics and different laws in a systematic manner.

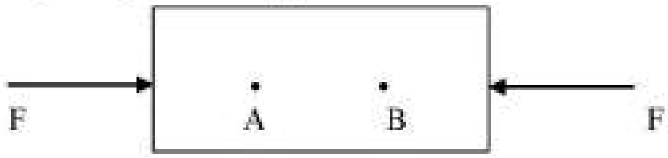


Concepts of: Physical quantity, Scalar quantity, and Vector quantity

1Particle: A particle is a body of infinitely small volume and the entire mass of the body is assumed to be concentrated at a point.

2.Rigid body: It is one, which does not alter its shape, or size or the distance between any two points on the body does not change on the application of external forces.

3.Deformable body: It is one, which alters its shape, or size or the distance between any two points on the body changes on the application of external forces.



In the above example, the body considered is rigid as long as the distance between the points A and B remains the same before and after application of forces, or else it is considered as a deformable body.

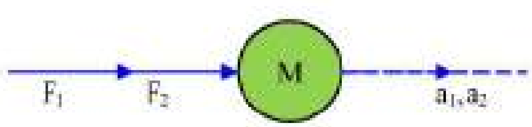
4.Force: According to Newton's I law, force is defined as an action or agent, which changes or tends to change the state of rest or of uniform motion of a body in a straight line.
Units of force: The gravitational (MKS) unit of force is the kilogram force and is denoted as **'kgf'. The absolute (SI) unit of force is the Newton and is denoted as 'N'.**

Note: 1 kgf = 'g' N (But g = 9.81m/s²) Therefore 1 kgf = 9.81 N or ≅ 10 N.

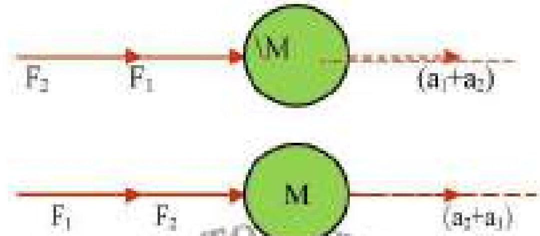
5.Continuum: The concept of continuum is purely theoretical or imaginary. Continuum is said to be made up of infinite number of molecules packed in such a way that, there is no gap between the molecules so that property functions remain same at all the points.

6.Point force: The concept of point force in purely theoretical or imaginary, here the force is assumed to be acting at a point or over infinity small area.

7.Principle physical independence of forces: Action of forces on bodies are independent, in other words the action of forces on a body is not influenced by the action of any other force on the body.



8.Principle of superposition of forces: Net effect of forces applied in any sequence on a body is given by the algebraic sum of effect of individual forces on the body.



9.Principle of transmissibility of forces: The point of application of a force on a rigid body can be changed along the same line of action maintaining the same magnitude and direction without affecting the effect of the force on the body.

Limitation of principle of transmissibility: Principle of transmissibility can be used only for rigid bodies and cannot be used for deformable bodies

Assumptions made in Engineering Mechanics

i) All bodies are rigid.

ii) Particle concept can be used wherever applicable.

iii) Principle of physical independence of forces is valid.

iv) Principle of superposition of forces is valid.

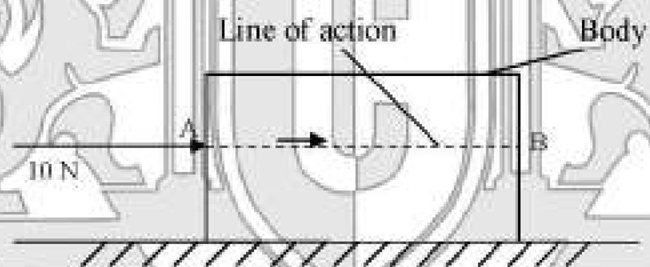
1.1 Characteristics of a force

These are ones, which help in understanding a force completely, representing a force and also distinguishing one force from one another.

A force is a vector quantity. It has four important characteristics, which can be listed as follows.

1) **Magnitude**: It can be denoted as 10 kgf or 100 N.
2) **Point of application**: It indicates the point on the body on which the force acts.
3) **Line of action**: The arrowhead placed on the line representing the direction represents it.
4) **Direction**: It is represented by a co-ordinate or cardinal system.

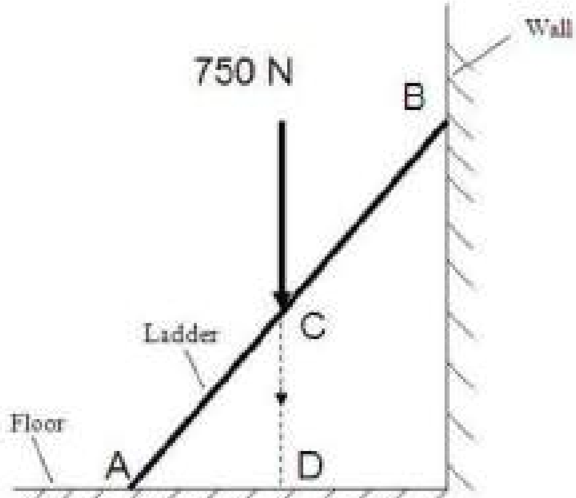
Ex.1: Consider a body being pushed by a force of 10 N as shown in figure below.



The characteristics of the force acting on the body are

1) Magnitude is 10 N.
2) Point of application is A.
3) Line of action is A to B or AB.
4) Direction is horizontally to right.

Ex.2: Consider a ladder AB resting on a floor and leaning against a wall, on which a person weighing 750 N stands on the ladder at a point C on the ladder.



The characteristics of the force acting on the ladder are
1) Magnitude is 750 N.
2) Point of application is C.
3) Line of action is C to D or CD.
4) Direction is vertically downward.

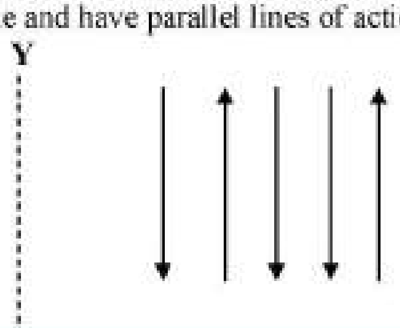
Idealization or assumptions in Mechanics: In applying the principles of mechanics to practical problems, a number of ideal conditions are assumed. They are as follows.
1) A body consists of continuous distribution of matter.
2) The body considered is perfectly rigid.
3) A particle has mass but not size.
4) A force acts through a very small point.

Classification of force systems: Depending upon their relative positions, points of applications and lines of actions, the different force systems can be classified as follows.

1) **Collinear forces**: It is a force system, in which all the forces have the same line of action.

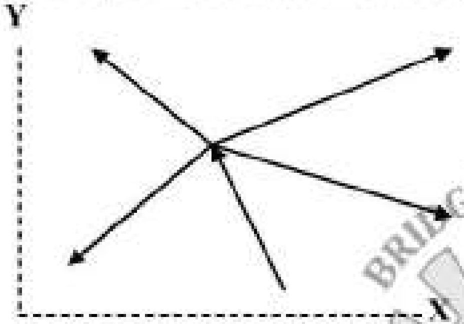
Ex.: Forces in a rope in a tug of war.

2) **Coplanar parallel forces**: It is a force system, in which all the forces are lying in the same plane and have parallel lines of action.



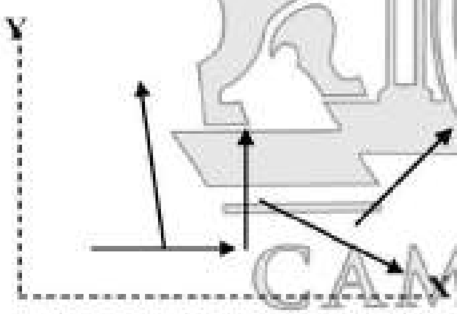
Ex.: The forces or loads and the support reactions in case of beams.

3) Coplanar Concurrent forces: It is a force system, in which all the forces are lying in the same plane and lines of action meet a single point.



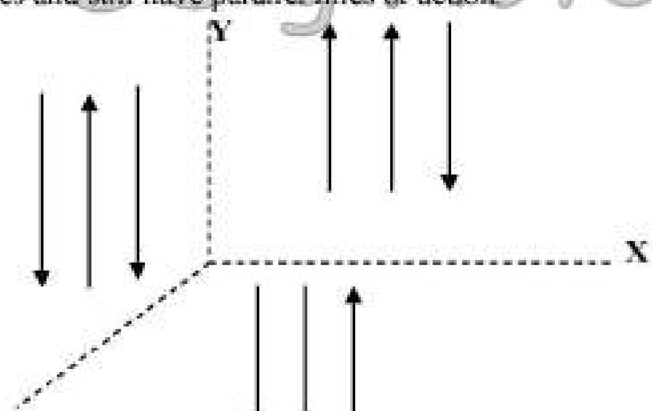
Ex.: The forces in the rope and pulley arrangement.

4) **Coplanar non-concurrent forces:** It is a force system, in which all the forces are lying in the same plane but lines of action do not meet a single point.



Ex.: Forces on a ladder and reactions from floor and wall, when a ladder rests on a floor and leans against a wall.

5) **Non- coplanar parallel forces**: It is a force system, in which all the forces are lying in the different planes and still have parallel lines of action.



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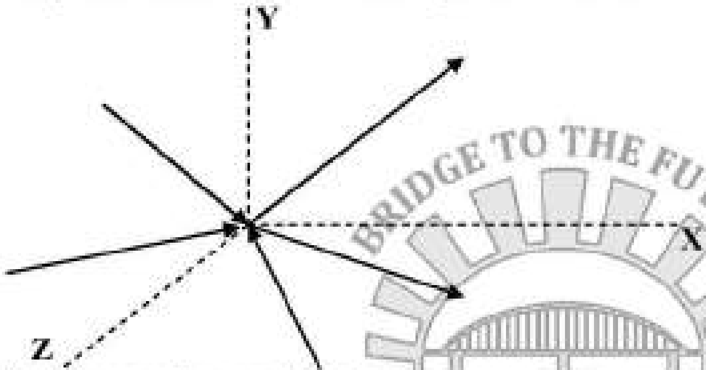
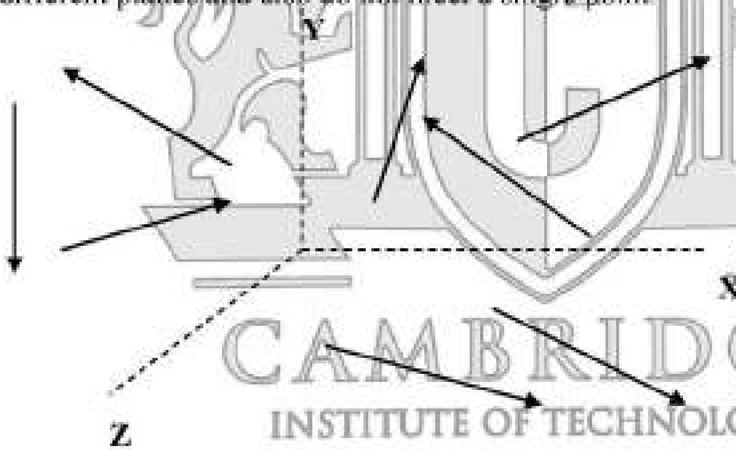
Ex: The forces acting and the reactions at the points of contact of bench with floor in a classroom.

6) **Non- coplanar concurrent forces** It is a force system, in which all the forces are lying in the different planes and still have common point of action.



Ex.: The forces acting on a tripod when a camera is mounted on a tripod.

7) **Non- coplanar non-concurrent forces**: It is a force system, in which all the forces are lying in the different planes and also do not meet a single point.



Ex.: Forces acting on a building frame.

2.3 Fundamental Laws in Mechanics

Following are considered as the fundamental laws in Mechanics.

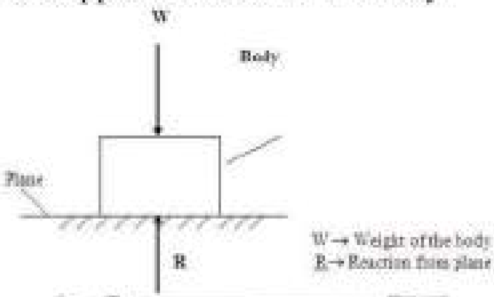
1) Newton's I law
2) Newton's II law
3) Newton's III law
4) Principle or Law of transmissibility of forces
5) Parallelogram law of forces.

1) **Newton's I law:** It states, "Every body continues in its state of rest or of uniform motion in a straight line, unless it is compelled to do so by force acting on it."

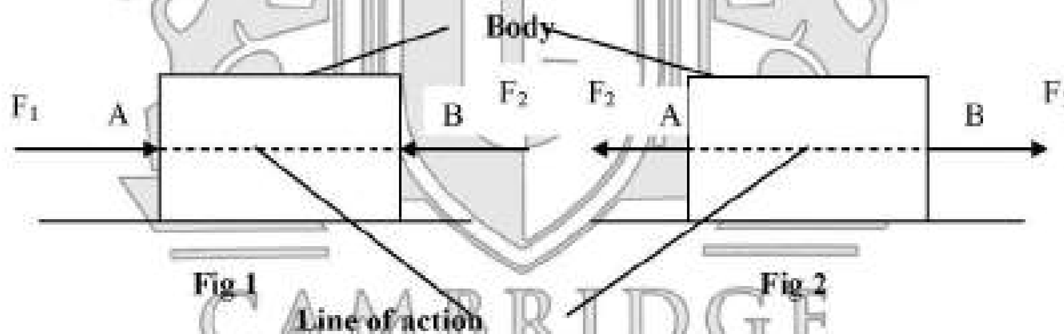
This law helps in defining a force.

2) <u>Newton's II law:</u> It states, "The rate of change of momentum is directly proportional to the **applied force and takes place in the direction of the impressed force."**
This law helps in defining a unit force as one which produces a unit acceleration in a body of unit mass, thus deriving the relationship F = m . a

3) <u>Newton's III law:</u> It states, **"For every action there is an equal and opposite reaction."**
The significance of this law can be understood from the following figure.
Consider a body weighing W resting on a plane. The body exerts a force W on the plane and in turn the plane exerts an equal and opposite reaction on the body.

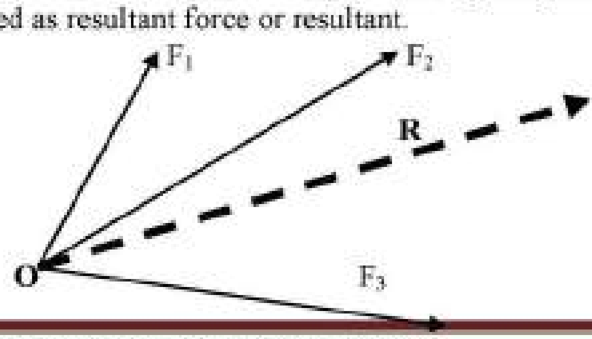


<u>Explanation of limitation</u>



In the example if the body considered is deformable, we see that the effect of the two forces on the body are not the same when they are shifted by principle of transmissibility. In the first case the body tends to compress and in the second case it tends to elongate. Thus principle of transmissibility is not applicable to deformable bodies or it is applicable to rigid bodies only.

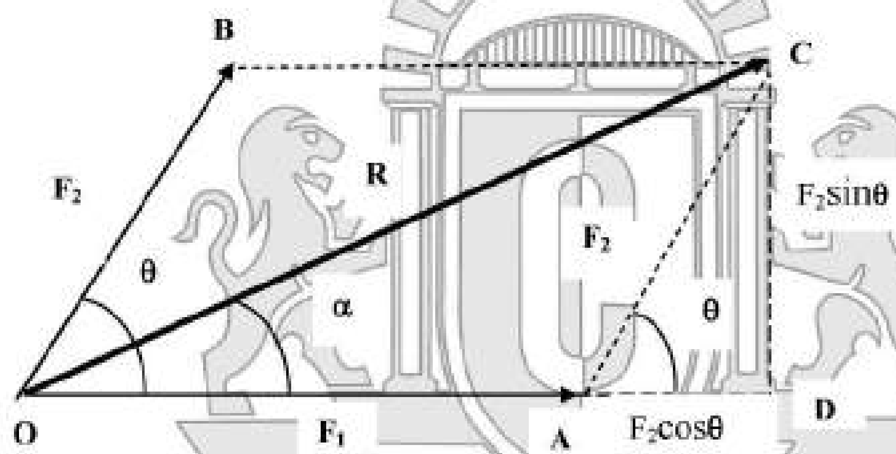
<u>Resultant Force:</u>
Whenever a number of forces are acting on a body, it is possible to find a single force, which can produce the same effect as that produced by the given forces acting together. Such a single force is called as resultant force or resultant.



In the above figure R can be called as the resultant of the given forces F_1, F_2 and F_3.
The process of determining the resultant force of a given force system is known as **Composition of forces.**

The resultant force of a given force system can be determining by **Graphical** and **Analytical** methods. In **analytical** methods two different principles namely: **Parallelogram law** of forces and **Method of Resolution** of forces are adopted.

Parallelogram law of forces: This law is applicable to determine the resultant of two coplanar concurrent forces only. This law states ***"If two forces acting at a point are represented both in magnitude and direction by the two adjacent sides of a parallelogram, then the resultant of the two forces is represented both in magnitude and direction by the diagonal of the parallelogram passing through the same point."***



Let F_1 and F_2 be two forces acting at a point O and θ be the angle between them. Let OA and OB represent forces F_1 and F_2 respectively both in magnitude and direction. The resultant R of F1 and F_2 can be obtained by completing a parallelogram with OA and OB as the adjacent sides of the parallelogram. The diagonal OC of the parallelogram represents the resultant R both magnitude and direction.

From the figure $OC = \sqrt{OD^2 + CD^2}$

$$= \sqrt{(OA + AD)^2 + CD^2}$$

$$= \sqrt{(F_1 + F_2 \cos\theta)^2 + (F_2 \sin\theta)^2}$$

$$\text{i.e } R = \sqrt{F_1^{\,2} + F_2^{\,2} + 2.F_1.F_2.\cos\theta} \quad \text{---------> 1}$$

Let α be the inclination of the resultant with the direction of the F1, then

$$\alpha = \tan^{-1}\left[\ F_2 \sin\theta \ \right] \quad \text{-------> 2}$$

$\overline{F_1 + F_2.\cos\theta}$

Equation 1 gives the magnitude of the resultant and Equation 2 gives the direction of the resultant.

Different cases of parallelogram law:

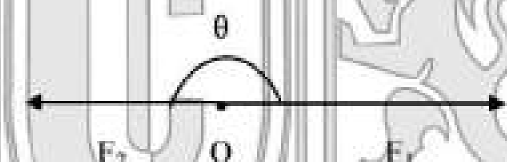
For different values of θ, we can have different cases such as follows:

Case 1: When $\theta = 90^0$:

$R = \sqrt{F_1^2 + F_2^2}$

$\alpha = \tan^{-1}\left[\frac{F_2}{F_1}\right]$

Case 2: When $\theta = 180^0$:



$R = [\, F_1 - F_2 \,]$

$\alpha = 0^0$

Case 3: When $\theta = 0^0$:

$R = [\, F_1 + F_2 \,]$

$\alpha = 0^0$

Example *Determine the magnitude of the resultant of the two forces of magnitude 12 N and 9 N acting at a point if the angle between the two forces is 30°.*

Solution:

$$P = 12 \text{ N}$$

$$Q = 9 \text{ N}$$

$$\theta = 30$$

$$\text{Resultant } R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{(12)^2 + (9)^2 + 2 \times 12 \times 9 \cos 30}$$

$$R = 20.29 \text{ N}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$= \frac{9\sin 30}{12 + 9\cos 30}$$

$$\tan\alpha = 0.2273$$

$$\boldsymbol{\alpha = 12.81.}$$

Example *Find the magnitude of two equal forces acting at a point with an angle of 60° between them, if the resultant is equal to $30\sqrt{3}$ N.*

Solution:

$$P = Q = F$$

$$R = 30\sqrt{3}$$

$$\theta = 60°$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$30\sqrt{3} = \sqrt{2F^2 + 2F^2\cos 60}$$

$$= F\sqrt{2(1 + \cos 60)}$$

$$F = \frac{30\sqrt{3}}{\sqrt{3}}$$

$$\mathbf{F = 30\ N.}$$

Example *The resultant of two forces when they act at right angles is 10 N. Whereas when they act at an angle of 60° the resultant is $\sqrt{145}$. Determine the magnitude of the forces.*

Solution:

Case (*i*) $R = 10$ N when $\theta = 90°$

Case (*ii*) $R = \sqrt{148}$ $\theta = 60°$

We know

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

when $\theta = 90°$

$$R = \sqrt{P^2 + Q^2}$$

$$10 = \sqrt{P^2 + Q^2}$$

$$10^2 = P^2 + Q^2$$

$$100 = P^2 + Q^2 \quad ...(i)$$

when $\theta = 60°$

$$\sqrt{148} = \sqrt{P^2 + Q^2 + 2PQ \cos 60}$$

$$= \sqrt{P^2 + Q^2 + 2PQ\,(0.5)}$$

$$148 = P^2 + Q^2 + PQ \quad ...(ii)$$

From (*i*)

$$148 = 100 + PQ$$

$$PQ = 48$$

$$2PQ = 96 \quad ...(iii)$$

Adding (*i*) and (*iii*)

$$100 + 96 = P^2 + Q^2 + 2PQ$$

$$196 = (P + Q)^2$$

$$P + Q = \sqrt{196}$$

$$P + Q = 14$$

$$P = 14 - Q \quad ...(iv)$$

From (*iii*)

$$2Q\,(14 - Q) = 96$$

$$28Q - 2Q^2 = 96$$

$$2Q^2 - 28Q + 96 = 0$$

$$Q^2 - 14Q + 48 = 0$$

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1,\ b = -14,\ c = 48$$

$$Q = \frac{+14 \pm \sqrt{(-14)^2 - 4 \times (1)\,(48)}}{2(1)}$$

$$= \frac{14 \pm \sqrt{4}}{2}$$

$$= \frac{14 \pm 2}{2}$$

$$Q_2 = \frac{14-2}{2} = \frac{12}{2} = 6\text{ N}$$

From (*iv*)

$$P = 14 - Q$$
$$P_1 = 14 - 8 = 6$$
$$P_2 = 14 - 6 = 8$$

∴ The forces are

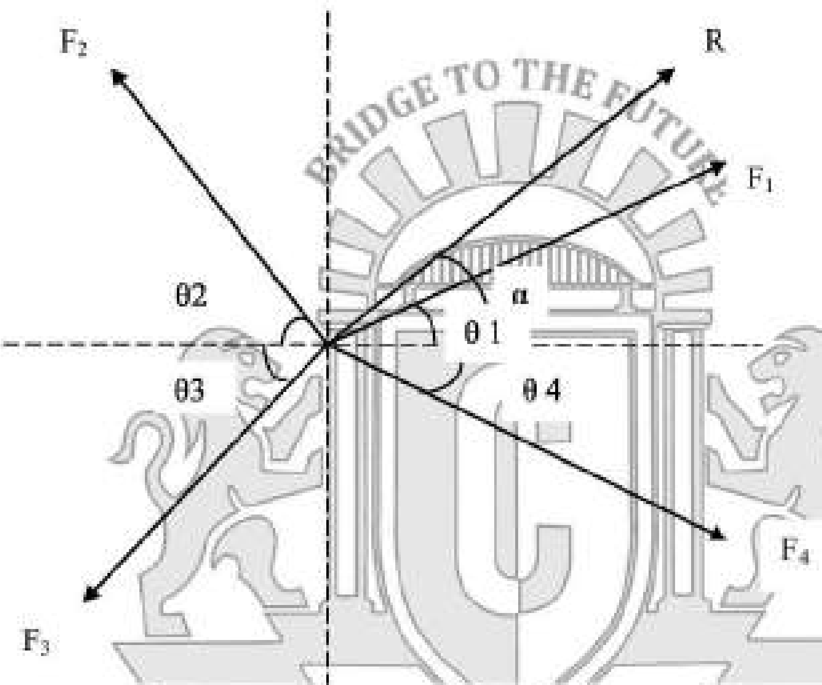
P = 8 N Q = 6 N.

$$Q_1 = \frac{14+2}{2} = \frac{16}{2} = 8\text{ N}$$

Composition of forces by method of Resolution

Introduction

If two or more forces are acting in a single plane and passing through a single point , such a force system is known as a



coplanar concurrent force system

Let F1, F2, F3 , F4 represent a coplanar concurrent force system. It is required to determine the resultant of this force system.

It can be done by first resolving or splitting each force into its component forces in each direction are then algebraically added to get the sum of component forces.

These two sums are then combines using parallelogram law to get the resultant of the force systems.

In the ∑fig, let fx_1, fx_2, fx_3, fx_4 be the components of Fx_1, Fx_2, Fx_3, Fx_4 be the forces in the X-direction.

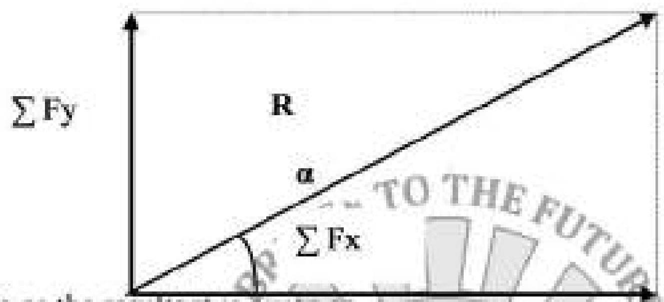
Let ∑ Fx be the algebraic sum of component forces in an x-direction

$$\sum Fx = fx_1 + fx_2 + fx_3 + fx_4$$

Similarly,

$\sum Fy = fy_1 + fy_2 + fy_3 + fy_4$

By parallelogram law,



The magnitude os the resultant is given as

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

The direction of resultant can be obtained if the angle α made by the resultant with x direction is determined here,

$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

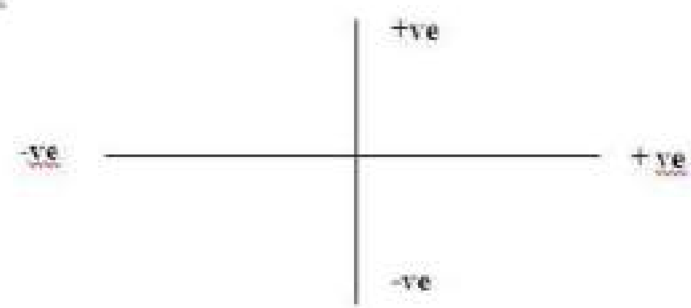
The steps to solve the problems in the coplanar concurrent force system are, therefore as follows.

1. Calculate the algebraic sum of all the forces acting in the x- direction (ie. $\sum Fx$) and also in the y- direction (ie. $\sum Fy$)
2. Determine the direction of the resultant using the formula

   $R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$
3. Determine the direction of the resultant using the formula

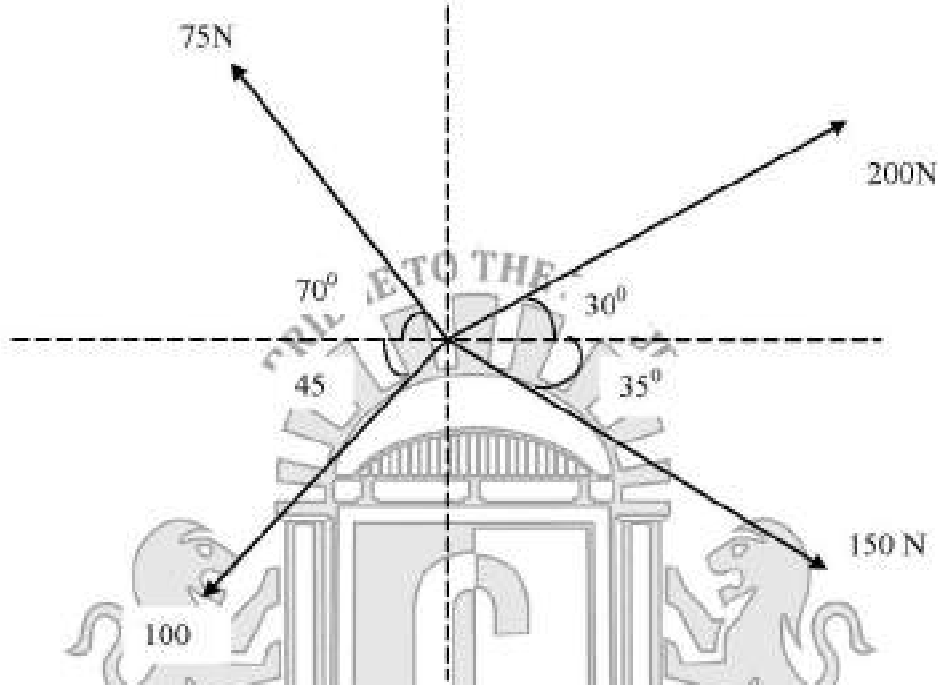
   $\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

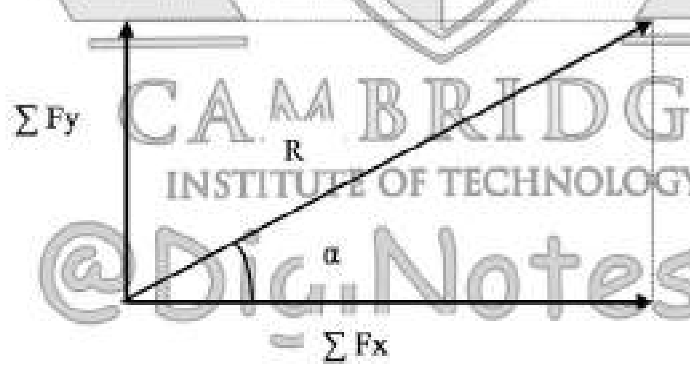
Sign Conventions:



3.1.1 Problems

Determine the magnitude & direction of the resultant of the coplanar concurrent force system shown in figure below.





Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$$R=\sqrt{(\sum Fx)^2+(\sum Fy)^2} \quad \text{and} \quad \alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

$\sum Fx = 200\cos 30^0 - 75\cos 70^0 - 100\cos 45^0 + 150\cos 35^0$

$\sum Fx = 199.7N$

$\sum Fy = 200\sin 30^0 + 75\sin 70^0 - 100\sin 45^0 - 150\sin 35^0$

$\sum Fy = 13.72\ N$

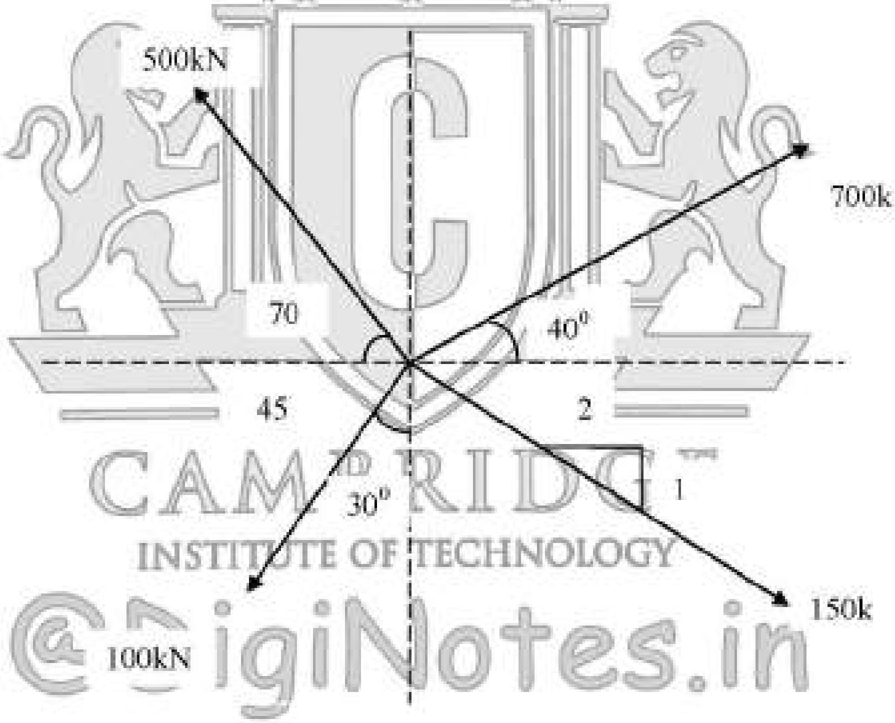
$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

R= 200.21N

$\alpha = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$\alpha = \tan^{-1}(13.72/\ 199.72) = 3.93^0$

Determine the resultant of the concurrent force system shown in figure.



Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$ and $\alpha = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$\sum Fx = 700\cos 40^0 - 500\cos 70^0 - 800\cos 60^0 + 200\cos 26.56^0$

$\sum Fx = 144.11$ kN

$\sum Fy = 700\sin 40^0 + 500\sin 70^0 - 800\sin 60^0 - 200\sin 26.56^0$

$\sum Fy = 137.55$kN

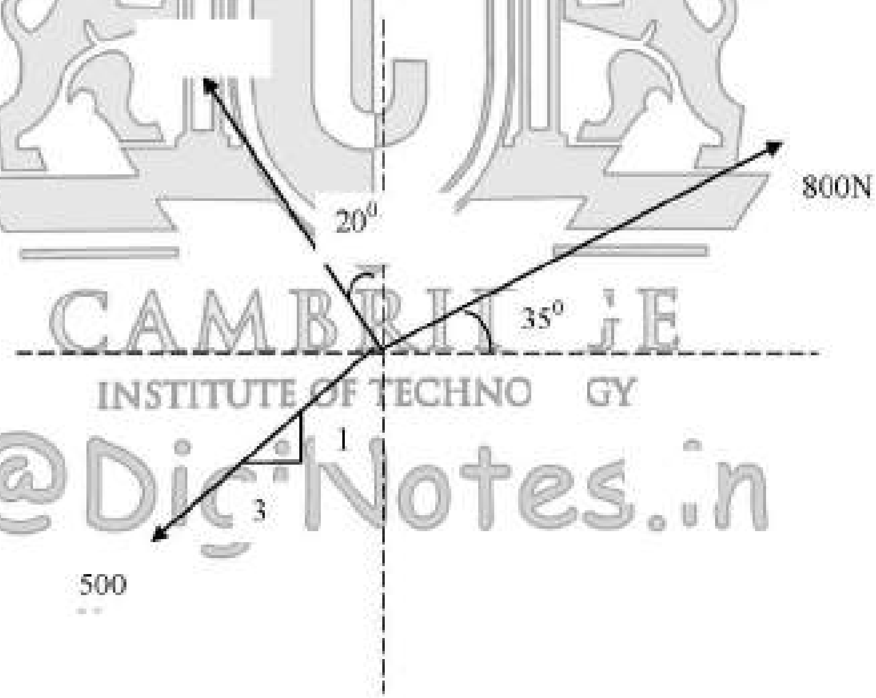
$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

R= 199.21N

$\alpha = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$\alpha = \tan^{-1}(137.55/ 144.11) = 43.66$

3. Determine the resultant of a coplanar concurrent force system shown in figure below



Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$R= \sqrt{(\sum Fx)^2+(\sum Fy)^2}$ and $\alpha = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$\sum Fx = 800\cos 35^0 - 100\cos70^0 + 500\cos60^0 + 0$

$\sum Fx = 1095.48$ N

$\sum Fy = 800\sin 35^0 + 100\sin70^0 + 500\sin60^0 - 600$

$\sum Fy = 110.90$ N

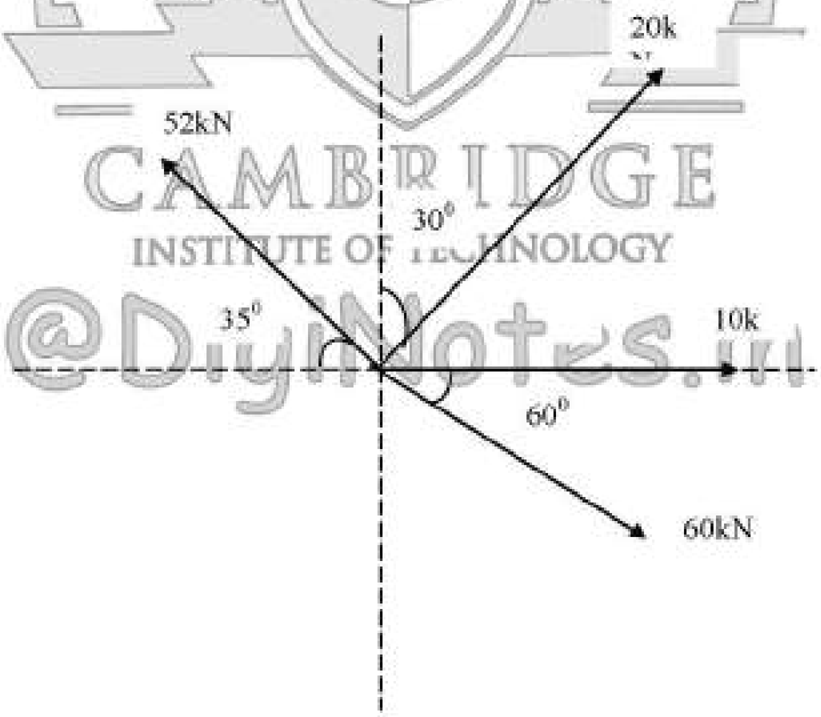
$R= \sqrt{(\sum Fx)^2+(\sum Fy)^2}$

R= 1101.08 N

$\alpha = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$\alpha = \tan^{-1}(110.90/ 1095.48) = 5.78^0$

4. The Magnitude and direction of the resultant of the resultant of the coplanar concurrent force system shown in figure.



Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$$R= \sqrt{(\sum Fx)^2 + (\sum Fy)^2} \text{ and } \alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

$\sum Fx = 20\cos 60^0 - 52\cos 30^0 + 60\cos 60^0 + 10$

$\sum Fx = 7.404$ kN

$\sum Fy = 20 \sin 60^0 + 52\sin 30^0 - 60\sin 60^0 + 0$

$\sum Fy = -8.641$ kN

$R= \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

R= 11.379 N

$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

$\alpha = \tan^{-1}(-8.641 / 7.404) = -49.40$

---

5. Determine the Magnitude and direction of the resultant of the resultant of the coplanar concurrent force system shown in figure.

$\theta_1 = \tan^{-1}(1/2) = 26.57$

$\theta_2 = 53.13$

Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$$R= \sqrt{(\sum Fx)^2 + (\sum Fy)^2} \text{ and } \alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

$\sum Fx = 200\cos 26.57^0 - 400\cos 53.13^0 - 50\cos 60^0 + 100\cos 50^0$

$\sum Fx = -21.844$ kN

$\sum Fy = 200 \sin 26.57^0 + 400\sin 53.13^0 - 50\sin 60^0 - 100 \sin 50^0$

$\sum Fy = 289.552$ kN

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

R= 290.374 N

$\alpha = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$\alpha = \tan^{-1}(289.552 / 21.844) = -85.68$

6. A hook is acted upon by 3 forces as shown in figure. Determine the resultant force on the hook.

Let R be the given resultant force system

$\sum Fx = ?$

$\sum Fy = ?$

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$ and $\alpha = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$\sum Fx = 80 \cos 25^0 + 50\cos 50^0 + 10\cos 45^0$

$\sum Fx = 111.71$ kN

$\sum Fy = 80 \sin 25^0 + 50\sin 50^0 - 10\cos 45^0$

$\sum Fy = 65.04$ kN

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

R= 129.26 N

$\alpha = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$\alpha = \tan^{-1}(65.04 / 111.71) = 30.20$

7. Two forces are acting on a structure at a point ' O" , as shown in fig. Determine the resultant force on the structure.

Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

In Δ^{le} AOC

Cos 60^0= AC/ 6

AC = $6\cos60^0$

AC =3 m, BC= 6m

In Δ^{le} AOC

Sin 60^0= OC/ 6

OC = $6\sin60^0$

OC = 5.196m

In Δ^{le} OBC,

$\theta = \tan^{-1}(OC/BC)$

$= \tan^{-1}(5.19 / 6) = 40.89^0$

$\sum Fx = 800 - 600 \cos 40.89^0$

$\sum Fx = 346.41$ N

$\sum Fy = 0 - 600 \sin 40.89^0$

$\sum Fy = -392.76$ kN

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

R= 523.7 N

$\alpha = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$\alpha = \tan^{-1}(-392.76 / 346.41) = 48.58$

Note:

From the above two figures, we can write

$\sum Fx = R \cos \alpha$

i.e The algebraic sum of all horizontal component forces is equal to the horizontal component of the resultant.

$\sum Fy = R \sin \alpha$

i.e The algebraic sum of all vertical component forces is equal to the vertical component of the resultant.

8. Two forces of magnitude 500N & 100N are acting at a point as shown in fig below. Determine the magnitude & Direction of third unknown force, such that the resultant of all the three forces has a magnitude of 1000N, making an angle of 45^0 as shown.

Let F_3 be the required third unknown force, which makes angle θ_3 with x- axis as shown

$F_3 = ?$ $\theta_3 = ?$

We know that

$R \cos \alpha = \sum Fx$

$1000 \cos 45^0 = 500 \cos 30^0 + 1000 \cos 60^0 + F_3 \cos \theta_3$

$F_3 \cos \theta_3 = -225.906N$ ------------- (1)

$R \sin \alpha = \sum Fy$

$1000 \sin 45^0 = 500 \sin 30^0 + 1000 \sin 60^0 + F_3 \sin \theta_3$

$F_3 \sin \theta_3 = -408.91N$ --------------- (2)

Dividing the Equation (2) by (1)

i.e. $F_3 \sin \theta_3 / F_3 \cos \theta_3 = -408.91 / -225.906$

$\tan \theta_3 = 1.810$

$\theta_3 = \tan^{-1}(1.810)$

$= 61.08$

From (1)

$F_3 \cos \theta_3 = -225.906N$

$F_3 = -225.906 / \cos 61.08 = -467.14$ N

Here , we have –ve values from both $F_3 \cos \theta_3$ and $F_3 \sin \theta_3$ (X & Y components of force F3). Thus the current direction for force F3 is represented as follows.

9. Two forces of magnitude 500N and 100N are acting at a point as shown in fig below. Determine the magnitude & direction of a 3rd unknown force such that the resultant of all the three forces has a magnitude of 1000N, making an angle of 450 & lying in the second quadrant.

F_3=? , θ_3=?

Let F_3 be a required third unknown force making an angle θ_3 with the x- axis to satisfy the given condition.

Let us assume F_3 to act as shown in fig.

We known that

$R \cos \alpha = \sum Fx$

$-1000\cos 45^0 = 500\cos 30^0 + 100\cos 60^0 + F_3 \cos \theta_3$

$F_3 \cos \theta_3 = -1190.119N$ ---------------- (1)

$R \sin \alpha = \sum Fy$

$1000\sin 45^0 = 500\sin 30^0 + 100\sin 60^0 + F_3 \sin \theta_3$

$F_3 \sin \theta_3 = 370.50N$ ---------------- (2)

Dividing the Equation (2) by (1)

i.e. $F_3 \sin \theta_3 / F_3 \cos \theta_3 = 370.50 / -1190.119$

$\tan \theta_3 = 0.3113$

$\theta_3 = \tan^{-1}(0.3113)$

$= 17.29$

From (2)

$F_3 \sin \theta_3 = 370.50N$

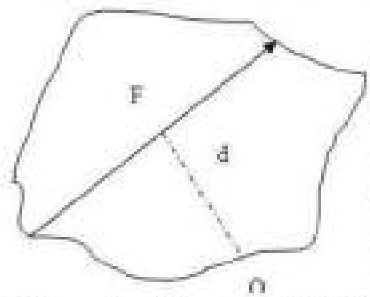
$F_3 = 370.50/\sin 17.29 = 1246.63N$

COMPOSITION OF COPLANAR NONCONCURRENT FORCE SYSTEM

If two or more forces are acting in a single plane, but not passing through the single point, such a force system is known as coplanar non concurrent force system.

Moment of Force:

It is defined as the rotational effect caused by a force on a body. Mathematically Moment is defined as the product of the magnitude of the force and perpendicular distance of the point from the line of action of the force from the point.

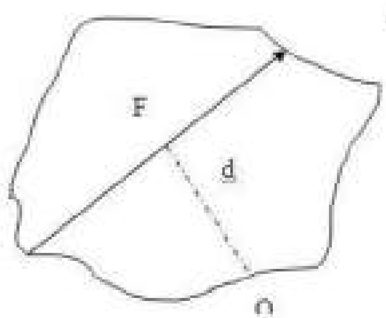


Let " F" be a force acting in a plane. Let" O" be a point or particle in the same plane. Let "d " be the perpendicular distance of the line of action of the force from the point "O" . Thus the moment of the force about the point "O" is given as

Mo= F x d

Moment or rotational effect of a force is a physical quantity dependent on the units for force and **distance. Hence the units for moment can be "Nm" or "KNm" or " N mm" etc.**

The moment produced by a force about differences points in a plane is different. This can be understood from the following figures.



Let " F" be a force in a plane and O_1, O_2, and O_3 be different points in the same plane
Let moment of the force " F" about point O_1 is Mo,
Mo_1= F x d_1
Let moment of the force " F" about point O_2 is Mo,
Mo_2= F x d_2

Let moment of the force " F" about point O_3 is Mo,
Mo= 0x F
The given force produces a clockwise moment about point O1 and anticlockwise moment about O_2 . A clockwise moment ( ) is treated as positive and an anticlockwise moment ( ) is treated as negative.

Note; The points O_1, O_2, O_3 about which the moments are calculated can also be called as moment centre.

Couple

Two forces of same magnitude separated by a definite distance, (acting parallely) in aopposite direction are said to form a couple.
A couple has a tendency to rotate a body or can produce a moment about the body. As such the moment due to a couple is also denoted as M.
Let us consider a point O about which a couple acts. Let S be the distance separating the couple. Let d1 & d2 be the perpendicular distance of the lines of action of the forces from the point o.

Thus the magnitude of the moment due to the couple is given a s
Mo = (Fx d1) + (F x d2)

Mo = F x d
i.e. The magnitude of a moment due to a couple is the product of force constituting the couple & the distance separating the couple . Hence the units for magnitude of a couple can be N m, kN m , N mm etc.

Varignon's principle of moments:

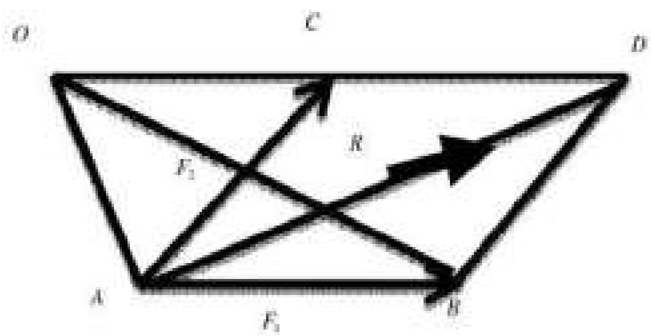
If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.
PROOF:
For example, consider only two forces F_1 and F_2 represented in magnitude and direction by AB and AC as shown in figure below.

Let O be the point, about which the moments are taken. Construct the parallelogram ABCD and complete the construction as shown in fig.

By the parallelogram law of forces, the diagonal AD represents, in magnitude and Direction, the resultant of two forces F_1 and F_2, let R be the resultant force.
By geometrical representation of moments
the moment of force about O=2 Area of triangle AOB
the moment of force about O=2 Area of triangle AOC
the moment of force about O=2 Area of triangle AOD

But,
Area of triangle AOD=Area of triangle AOC + Area of triangle ACD
Also, Area of triangle ACD=Area of triangle ADB=Area of triangle AOB
Area of triangle AOD=Area of triangle AOC + Area of triangle AOB

Multiplying throughout by 2, we obtain
2 Area of triangle AOD =2 Area of triangle AOC+2 Area of triangle AOB
i.e., Moment of force R about O=Moment of force F_1 about O + Moment of force F_2 about O
Similarly, this principle can be extended for any number of forces.



By using the principles of resolution composition & moment it is possible to determine Analytically the resultant for coplanar non-concurrent system of forces.
The procedure is as follows:

1. Select a Suitable Cartesian System for the given problem.
2. Resolve the forces in the Cartesian System
3. Compute $\sum fxi$ and $\sum fyi$
4. Compute the moments of resolved components about any point taken as the moment **Centre O. Hence find $\sum M0$**

$$R = \sqrt{\left(\Sigma f x_i\right)^2 + \left(\Sigma f y_i\right)^2} \qquad \alpha_R = \tan^{-1}\left(\frac{\sum f y_i}{\sum f x_i}\right)$$

5. Compute moment arm $d_R = \left|\frac{\sum M_o}{R}\right|$

6. Also compute x- intercept as $x_R = \left|\frac{\sum M_o}{\sum f_y}\right|$

7. And Y intercept as $y_R = \left|\frac{\sum M_o}{\sum f_x}\right|$

3.3.1.Problems

Example 1: Compute the resultant for the system of forces shown in Fig 2 and hence compute the Equilibriant.

$$\sum f_x = 44.8 - 32\cos 60^\circ$$
$$= 28.8 \text{ KN}$$

$$\sum f_y = 8 - 14.4 - 32\sin 60^\circ$$
$$= -34.11 \text{ KN}$$
$$R = 44.6 \text{ KN}$$
$$\alpha_R = 49.83^\circ$$
$$\circlearrowright + \sum M_o = -14.4(3) + 32\cos 60^\circ(4) - 32\sin 60^\circ(3)$$
$$= -62.34 KNM$$
$$d_R = \frac{62.34}{44.64} = 1.396 \text{ m}$$
$$x_R = \frac{62.34}{34.11} = 1.827 \text{ m}$$
$$y_R = \frac{62.34}{28.8} = 2.164 \text{ m}$$

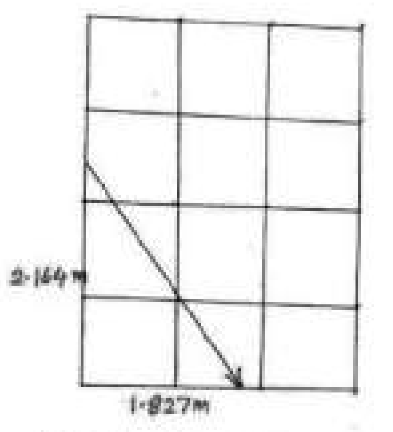


Fig. 2(b) Example 1

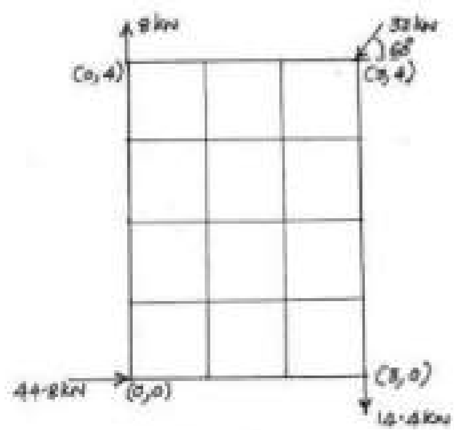


Fig. 2 Example 1

Example 2: Find the Equilibriant for the rigid bar shown in Fig 3 when it is subjected to forces.

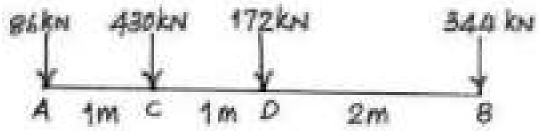


Fig. 3 Example 2

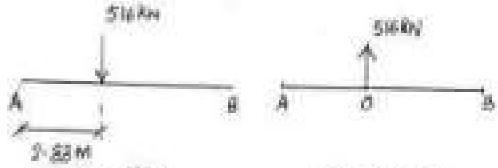


Fig. 3(a) Example 2

$$\circlearrowleft + \sum M_A = -430(1) + 172(2) - 344(4)$$
$$= -1462 \text{ KNM}$$

- Resultant and Equilibriant

$$\sum f_{x_i} = 0$$
$$\sum f_{y_i} = -516KN$$
$$\alpha_R = 90^\circ;$$

Example 3: A bar AB of length 3.6 m and of negligible weight is acted upon by a vertical force F1 = 336kN and a horizontal force F2 = 168kN shown in Fig 4. The ends of the bar are in contact with a smooth vertical wall and smooth incline. Find the equilibrium position of the bar by computing the angle θ.

$$\tan\alpha = 0.9/1.2$$
$$\alpha = 36.87^\circ$$

$$\sum f_x = 0$$
$$H_A - F_2 - R_B\cos 53.13^\circ = 0 \ldots\ldots\ldots(1)$$
$$\sum f_y = 0$$
$$R_B \sin 53.13^\circ - F_1 = 0$$
$$R_B = 420KN;$$

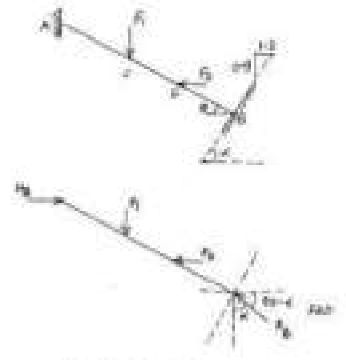


Fig 4 Example 3

- Eq. 1 gives HA=420 KN

$$\circlearrowleft + \sum M_B = 0;$$
$$-H_A(3.6\sin\theta) + 336(2.1\cos\theta) - 168(1.2\sin\theta) = 0$$
$$-1310.4\sin\theta + 705.6\cos\theta = 0$$
$$\tan\theta = 0.538$$
$$\theta = 28.3^\circ$$

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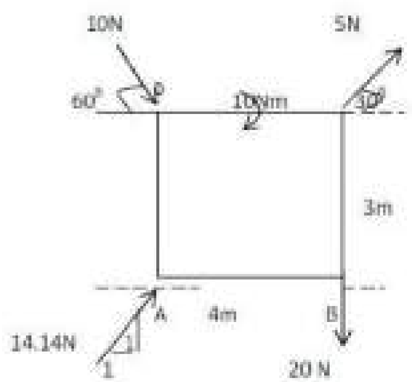
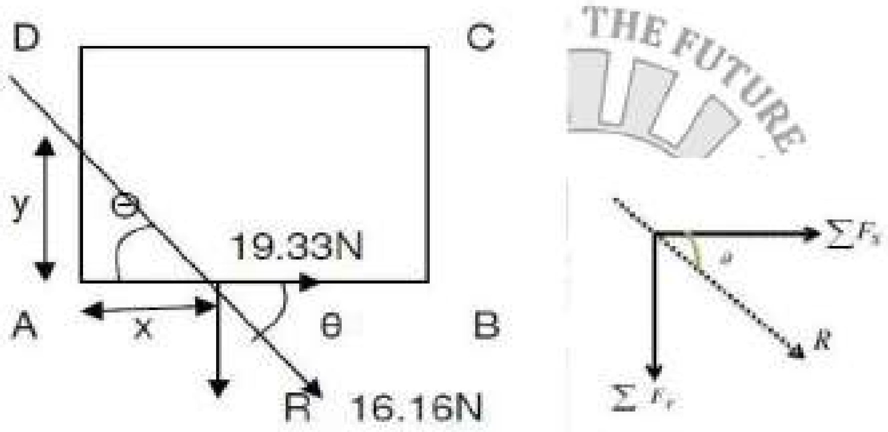
2. Determine the resultant of the force system acting on the plate. As shown in figure given below with respect to AB and AD.



$\sum Fx = 5\cos30^0 + 10\cos60^0 + 14.14\cos45^0$
$= 19.33N$
$\sum Fy = 5\sin30^0 - 10\sin60^0 + 14.14\sin45^0$
$= -16.16N$
$R = \sqrt{(\sum Fx^2 + \sum Fy^2)} = 25.2N$

$\theta = Tan^{-1}(\sum Fy / \sum Fx)$
$\theta = Tan^{-1}(16.16/19.33) = 39.89^0$

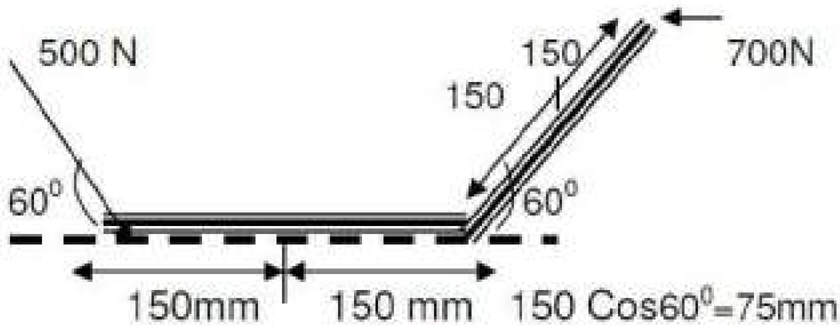


Tracing moments of forces about A and applying varignon's principle of moments we get
$+16.16X - 20x4 + 5\cos30^0x3 - 5\sin30^0x4 + 10 + 10\cos60^0x3$

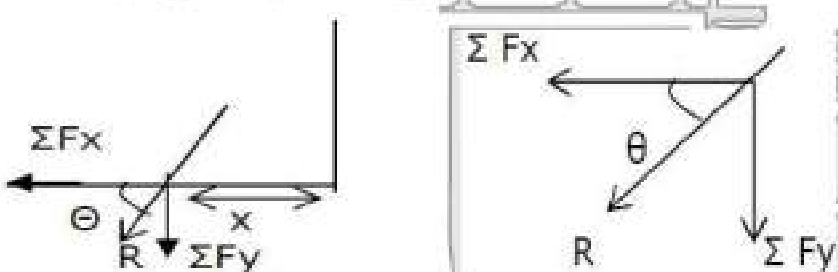
$x = 107.99/16.16 = 6.683m$

Also $\tan39.89 = y/6.83$
$y = 5.586m.$

3. The system of forces acting on a crank is shown in figure below. Determine the magnitude, direction and the point of application of the resultant force.



$\sum Fx = 500\cos 60^0 - 700$
$= 450N$
$\sum Fy = 500\sin 60^0$
$= -26.33N$
$R = \sqrt{(\sum Fx^2 + \sum Fy^2)} = (-450)^2 + (-2633)^2$
R = 267.19N (Magnitude)

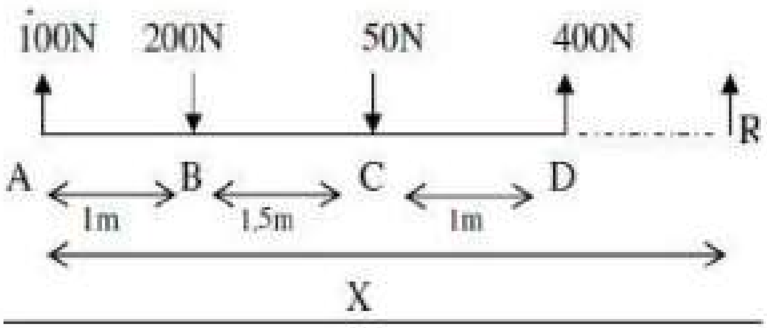


$\theta = \tan^{-1}(\sum Fy/ \sum Fx)$
$\theta = \tan^{-1}(2633/450)$
$\theta = 80.30^0$ (Direction)

Tracing moments of forces about O and applying varignon's principle of moments we get

$-2633x\ x = -500x\ \sin 60^0 x300-1000x150+1200x150\cos 60^0 -700x300\sin 60^0$
x = -371769.15/-2633
x= 141.20mm from O towards left (position).

4.For the system of parallel forces shown below, determine the magnitude of the resultant and also its position from A .

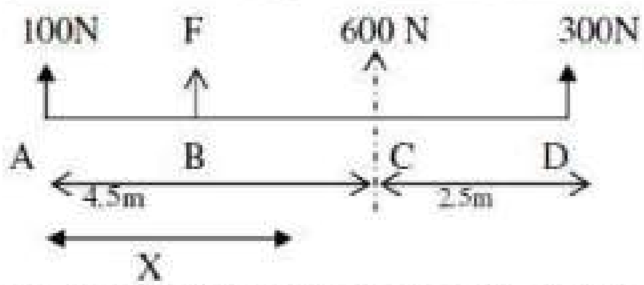


$\sum Fy = +100 -200 -50 +400$
$= +250N$
ie. $R = \sum Fy = 250N$ (↑)

Since $\sum Fx = 0$

Taking moments of forces about A and applying varignon's principle of moments
$-250\ x = -400 \times 3.5 + 50 \times 2.5 + 200 \times 1 - 100 \times 0$
$X = -1075/ -250 = 4.3m$

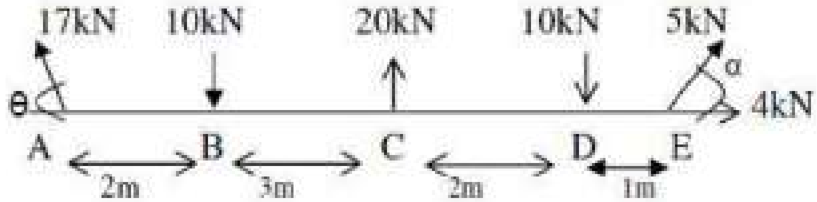
5. The three like parallel forces 100 N,F and 300 N are acting as shown in figure below. If the resultant R=600 N and is acting at a distance of 4.5 m from A ,find the magnitude of force F and position of F with respect to A



Let x be the distance from A to the point of application of force F
Here $R = \sum Fy$
$600=100+F+300$
$F = 200\ N$

Taking moments of forces about A and applying varignon's principle of moments,
We get
$600 \times 4.5 = 300 \times 7 + F\ x$
$200\ x = 600 \times 4.5 - 300 \times 7.$
$X = 600/200 = 3m$ from A

6. A beam is subjected to forces as shown in the figure given below. Find the magnitude, direction and the position of the resultant force.



Given tan θ= 15/8 sin θ = 15/ 17 cos θ = 8/17

tanα= 3/4 sin α = 3/5 cos α = 4/5

∑ Fx = 4 +5 cos α – 17 cos θ

= 4 + 5 x 4/5 – 17 x 8/17

∑ Fx = 0

∑ Fy = 5 sin α -10 + 20 – 10 + **17 sin θ**

= 5 x 3/5 -10+ 20 – 10 + 17 x 15/17

∑ Fy = 18 kN (↑)

Resultant force R = □□(∑Fx)^2 + (∑Fy)^2 = □□0+182

R = 18 kN(↑)

Let x = distance from A to the point of application R

Taking moments of forces about A and applying Varignon's theorem of moments

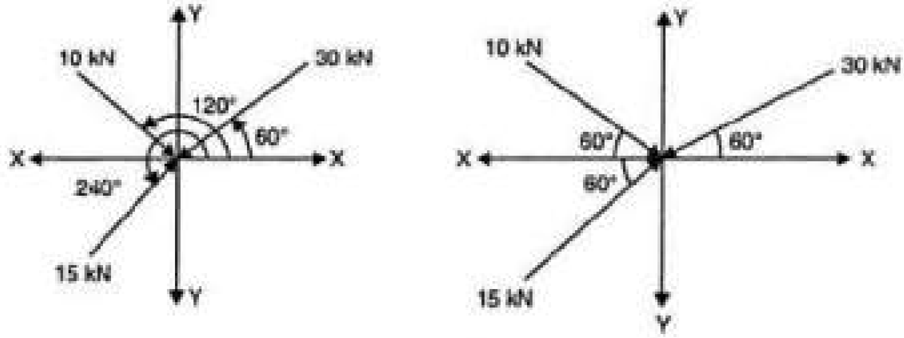
-18 x = -5 x sin α x 8 +10 x 7 -20 x 5 + 10 x 2

= -3 x 8 +10 x7 – 20 x 5 + 10 x 2

X = -34/-18 = 1.89m from A (towards left)

Example *Three forces of magnitude 30 kN, 10 kN and 15 kN are acting at a point O. The angles made by 30 kN force, 10 kN force and 15 kN force with x-axis are 60°, 120° and 240° respectively.*

Determine the magnitude and direction of the resultant force.



F.B.D

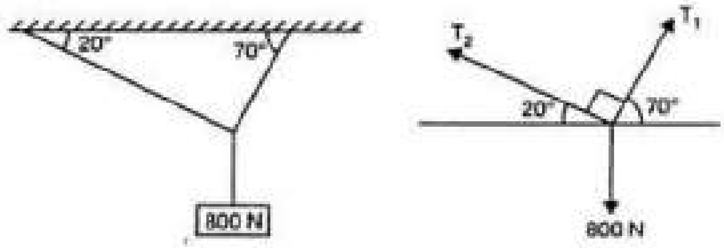
Solution:

$$\Sigma H = -30\cos 60 + 10\cos 60 + 15\cos 60$$
$$= -2.5 \text{ kN}$$
$$\Sigma V = -30\sin 60 - 10\sin 60 + 15\sin 60$$
$$= -21.65 \text{ kN}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-2.5)^2 + (21.65)^2}$$
$$= 21.79 \text{ kN}$$

$$\tan\alpha = \frac{\Sigma V}{\Sigma H} = \frac{-21.65}{-2.5} = 83°41'.$$

Example *A weight of 800 N is suspended by two chains as shown in figure. Determine the tensions in each chain.*



F.B.D.

Solution:

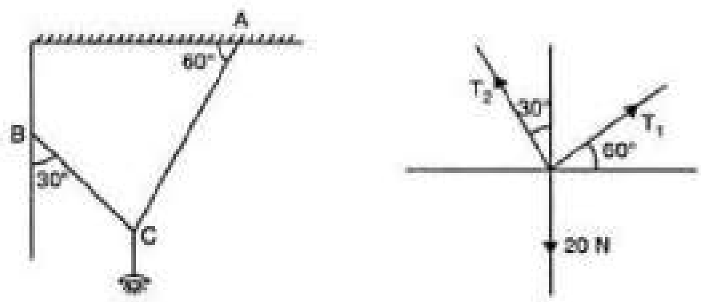
$$\Sigma H = 0$$
$$-T_2\cos 20 + T_1\cos 70 = 0$$
$$T_1\cos 70 = T_2\cos 20$$
$$T_2 = \frac{T_1\cos 70}{\cos 20}$$
$$T_2 = T_1\,(0.364) \qquad \ldots(i)$$

$$\Sigma V = 0$$
$$T_2 \sin 20 + T_1 \sin 70 = 800$$
$$\sin 20\ T_1\ (0.364) + T_1 \sin 70 = 800$$
$$1.3\ T_1 = 800$$
$$T_1 = 751.75\ \text{N}$$

From (*i*)

$$T_2 = 751.75\ (0.364)$$
$$\mathbf{T_2 = 273.64\ N.}$$

Example *An electric light fixture weighing 20 N hangs from a point C, by two strings AC and BC. AC is inclined at 60° to the horizontal and BC at 30° to the vertical as shown in Fig. Determine the forces in the strings AC and BC.*



F.B.D.

Solution:

$$\Sigma H = 0$$
$$-T_2 \sin 30 + T_1 \cos 60 = 0$$
$$T_2 \sin 30 = T_1 \cos 60$$
$$T_2 = T_1 \frac{\cos 60}{\sin 30}$$
$$T_2 = T_1 \frac{0.5}{0.5}$$
$$T_2 = T_1$$
$$\Sigma V = 0$$
$$T_2 \cos 30 + T_1 \sin 60 = 20$$
$$T_1 \cos 30 + T_1 \sin 60 = 20$$
$$1.73\ T_1 = 20$$

$$T_1 = \frac{20}{1.73} = \mathbf{11.547\ N}$$
$$T_1 = T_2 = 11.547\ \text{N}$$

Example *Two forces of magnitude 15 N and 12 N are acting at a point. If the angle between the two forces is 60°, determine the resultant of the forces in magnitude and direction.*

Solution:

$$P = 15\text{ N}$$
$$Q = 12\text{ N}$$
$$\theta = 60°$$

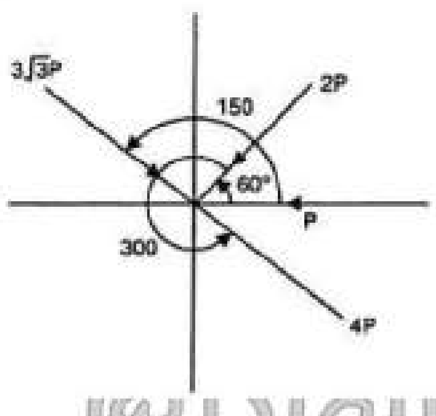
Resultant ⇒

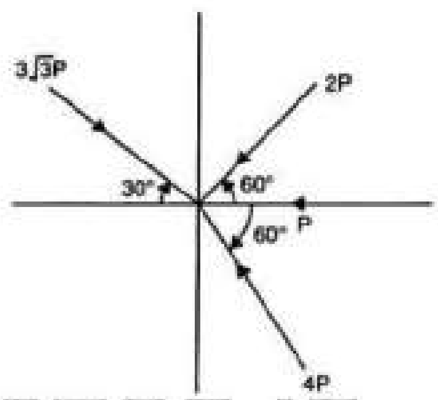
$$R = \sqrt{(15)^2 + (12)^2 + 2 \times 15 \times 12 \times \cos 60}$$
$$= 23.43\text{ N}$$

Direction ⇒

$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$
$$= \frac{12 \sin 60}{15 + 12 \cos 60}$$
$$= 0.495$$
$$\alpha = 26.31.$$

Example *Four forces of magnitude P, 2P, $3\sqrt{3}$ P and 4P are acting at a point O. The angles made by these forces with x-axis are 0°, 60°, 150° and 300° respectively. Find the magnitude and direction of the resultant force.*





Solution: $\Sigma H = -P - 2P\cos 60 + 3\sqrt{3}\ P\cos 30 - 4P\cos 60$

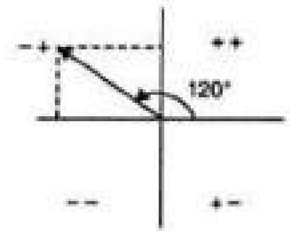
$$\Sigma H = \frac{P}{2} = 0.5\,P$$

$$\Sigma V = -2P\sin 60 - 3\sqrt{3}\ P\sin 30 + 4P\sin 60^{\circ}$$

$$= -0.87\ P$$

$$R = P\sqrt{(0.5)^2 + (-0.87)^2}$$

R = PN

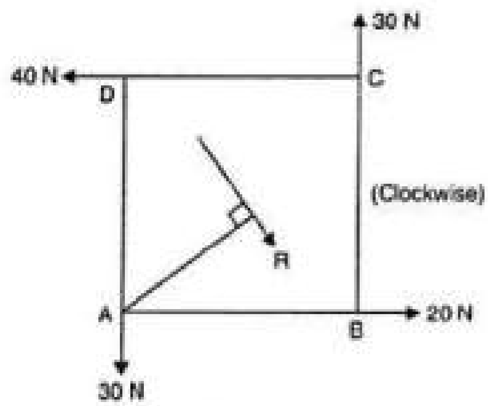


$$\tan\phi = \frac{\Sigma V}{\Sigma H} = \frac{-0.87}{0.5} = 1.74$$

$$\phi = 60.11$$

$$\phi = 180 - 60 = 120$$

Example *Four forces of magnitude 20 N, 30 N, 40 N and 50 N are acting respectively along four sides of a square taken in order. Determine the magnitude, direction and position of the resultant force.*



Solution:

$$\Sigma H = 20 - 40 = -20$$

$$\Sigma V = 30 - 50 = -20$$

$$R = \sqrt{(-20)^2 + (-20)^2}$$

$$R = 28.28\ \text{N}$$

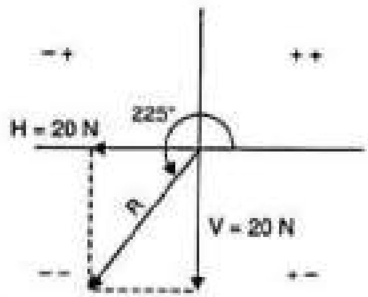
Direction of the resultant

$$\tan \phi = \frac{\Sigma V}{\Sigma H} = \frac{-20}{-20}$$

$$\tan \phi = 1$$

$$\phi = 45°$$

Since ΣH and ΣV are –ve ϕ has between 180° and 270 *i.e.*, $\phi = 180 + 45 = 225$.



Position of the resultant force :

The position of the resultant force is obtained by equating the clockwise moments and anticlockwise moment about A.

Let x = perpendicular distance between A and line of action of the resultant force

a = side of the square ABCD

Taking moments about A

$$-50 \times 0 - 20 \times 0 - 30 \times a - 40 \times a = R \times \text{Perpendicular distance of } R \text{ from } A$$

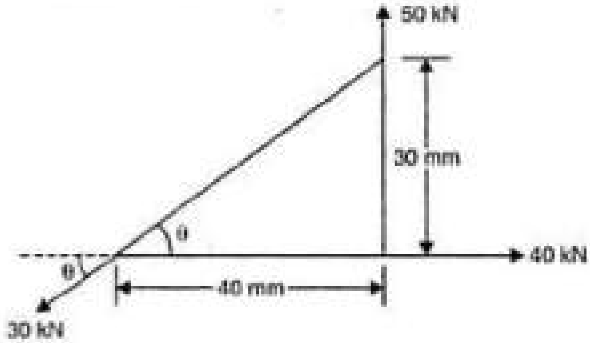
$$-30\,a - 40\,a = 28.28 \times x$$

$$-70\,a = 28.28 \times x$$

$$x = -\frac{70\,a}{28.28}$$

$$x = -2.47\,a \text{ (anticlockwise).}$$

Example *A triangle ABC has its sides AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 kN, 50 kN and 30 kN act along the sides AB, BC and CA respectively. Determine the resultant of such a system of forces.*



Solution:

From figure $\tan\theta = \frac{30}{40} = 0.75$

$$\theta = \tan^{-1} 0.75$$

$$\theta = 36^\circ.87'$$

$$\Sigma H = 40 - 30\cos\theta$$

$$= 40 - 30\cos 36^\circ.87'$$

$$\Sigma H = 16 \text{ kN}$$

$$\Sigma V = 50 - 30\sin\theta$$

$$= 50 - \sin 36^\circ.87' \times 30$$

$$\Sigma V = 32 \text{ kN}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2}$$

$$\mathbf{R = 35.78\ kN.}$$

Example *The four coplanar forces are acting at a point as shown in figure. Determine the resultant and direction of the resultant.*

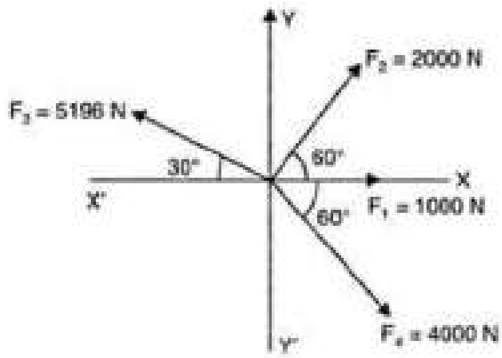
Solution:

$$\Sigma H = 1000 + 2000\cos 60 - 5196\cos 30 + 4000\cos 60$$

$$= -499.87 \text{ N}$$

$$\Sigma V = 2000\sin 60 + 5196\sin 30 - 4000\sin 60$$

$$= 865.95 \text{ N}$$

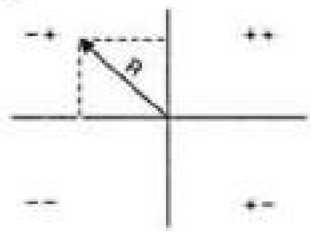


$$R = \sqrt{(-499.87)^2 + (865.95)^2}$$

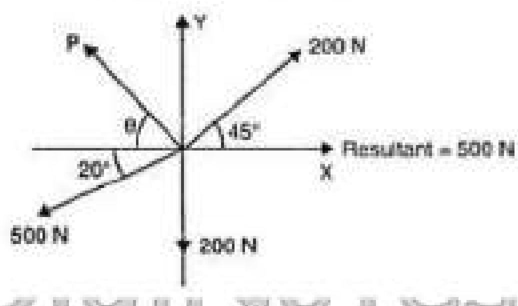
$$R = 1000\ \text{N}$$

$$\tan\phi = \frac{\sum V}{\sum H} = \frac{865.95}{-499.87} = 1.73$$

$\phi = 60°$.



Example *The four coplanar forces acting at a point as shown in figure one of the forces is unknown and its magnitude is shown by P. The resultant is having a magnitude of soon and is acting along x-axis. Determine the unknown force P and its inclination with x-axis.*



Solution: $\sum H = 200 \cos 45 - P \cos \theta - 500 \cos 20 + 500 = 0$

$P \cos \theta = 171.57$ ...(*i*)

$\sum V = 200 \sin 45 + P \sin \theta - 500 \sin 20 - 200 = 0$

$P \sin \theta = 500 \sin 20 + 200 - 200 \sin 45$

$P \sin \theta = 229.6$ ...(*ii*)

Squaring and adding (*i*) and (*ii*)

$$P^2(\sin^2 \theta + \cos^2 \theta) = (171.57)^2 + (229.6)^2$$

$$P^2 = 82152.4$$

$$P = \mathbf{286.61\ N}$$

(*ii*) ÷ (*i*)

$$\frac{P \sin \theta}{P \cos \theta} = \frac{229.6}{171.57} = 1.338$$

$$\tan \theta = 1.338 \quad \theta = \mathbf{53°.23'}.$$

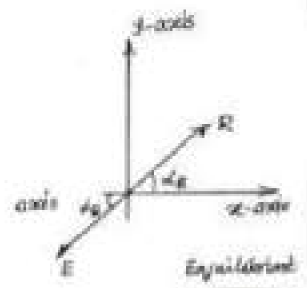
MODULE -2

EQUILIBRIUM OF FORCES

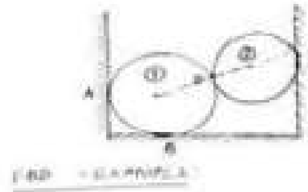
Equilibrium: Equilibrium is the status of the body when it is subjected to a system of forces. We know that for a system of forces acting on a body the resultant can be determined. By Newton's 2nd Law of Motion the body then should move in the direction of the resultant with some acceleration. If the resultant force is equal to zero it implies that the net effect of the system of forces is zero this represents the state of equilibrium. For a system of coplanar concurrent forces for the resultant to be zero, hence

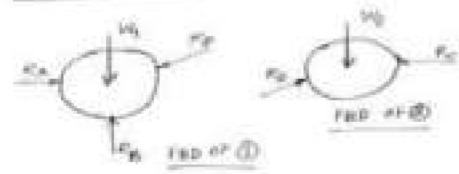
$$\sum f_{x_i} = 0$$
$$\sum f_{y_i} = 0$$

Equilibriant : Equilbriant is a single force which when added to a system of forces brings the status of equilibrium . Hence this force is of the same magnitude as the resultant but opposite in sense. This is depicted in Fig 4.



Free Body Diagram: Free body diagram is nothing but a sketch which shows the various forces acting on the body. The forces acting on the body could be in form of weight, reactive forces contact forces etc. An example for Free Body Diagram is shown below.



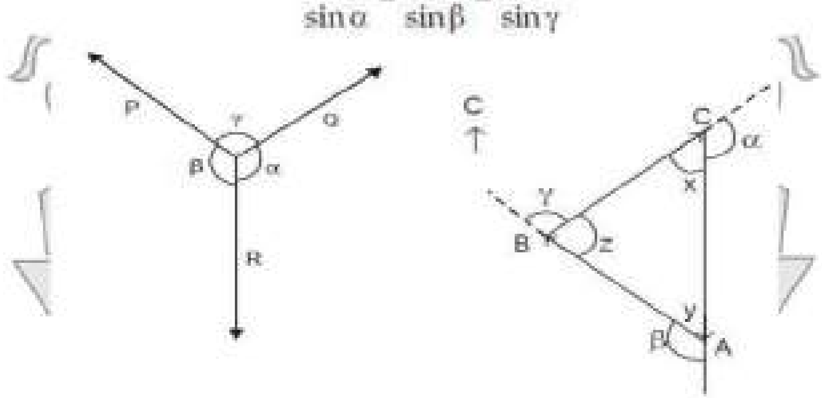


Lami's Theorem

If three forces acting on a particle keep it in equilibrium, each force is proportional to the sine of the angle between the other two.

P, Q and *R* three forces acting at a point keeping it in equilibrium, Fig. If α, β and γ are the angles apposite to each of them respectively,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



This law is a direct consequence of the triangle law. Since the forces are in equilibrium, they can be represented by the sides of the triangle *ABC* taken in order. A general property of any triangle is that each side is proportional to the sine of the angle opposite to it. Thus in the triangle *ABC* drawn with the sides parallel to the forces *P, Q,* and *R,*

$$\frac{AB}{\sin x} = \frac{BC}{\sin y} = \frac{CA}{\sin z}$$

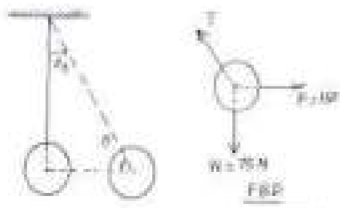
Here *x, y* and *z* are the angles of the triangle *ABC*. But by the triangle law of forces, the sides of the triangle are proportional to the respective force. From the Fig. 1.2

$$\sin x = \sin(180 - \alpha) = \sin\alpha$$
$$\sin y = \sin(180 - \beta) = \sin\beta$$
$$\sin z = \sin(180 - \gamma) = \sin\gamma$$

Hence $$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{Q}{\sin\gamma} = \text{constant}$$

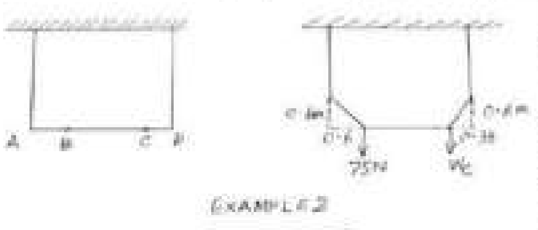
Thus, if 3 forces acting on a particle are in equilibrium, each force is proportional to the sine of the angle between the other two.

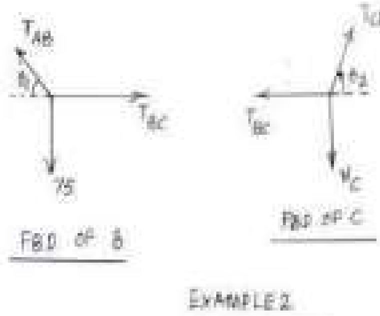
Example 1 : A spherical ball of weight 75N is attached to a string and is suspended from the ceiling. Compute tension in the string if a horizontal force F is applied to the ball. Compute the angle of the string with the vertical and also tension in the string if F =150N



$$\Sigma f_{x_1} = 0$$
$$f - T\cos\theta = 0$$
$$150 - T\cos\theta = 0$$
$$T\cos\theta = 150$$

Example 2: A string or cable is hung from a horizontal ceiling from two points A and D. The string AD, at two points B and C weights are hung. At B, which is 0.6 m from a weight of 75 N is hung, C, which is 0.35 m from D, a weight of wc is hung. Compute wc such that the string portion BC is horizontal.





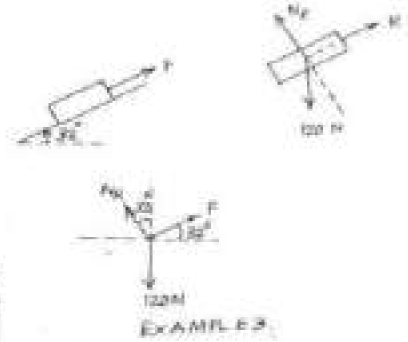
FBD of B

$$\sum f_{x_1} = 0$$
$$T_{BC} - T_{AB}\cos\theta_1 = 0$$
$$\sum f_{y_1} = 0$$
$$T_{AB}\sin\theta_1 - 75 = 0$$
$$T_{AB} = 75\sqrt{2}\,N, T_{BC} = 75N$$

FBD of C

$$\sum f_{x_1} = 0$$
$$-T_{BC} + T_{CD}\cos\theta_2 = 0$$
$$T_{CD} = 148.85N$$
$$\sum f_{y_1} = 0$$
$$T_{CD}\sin\theta_2 - W_c = 0$$
$$W_c = 128.57N$$

Example 3: A block of weight 120N is kept on a smooth inclined plane. The plane makes an angle of 320 with horizontal and a force F allied parallel to inclined plane. Compute F and also normal reaction.



- LAMI'S Theorem

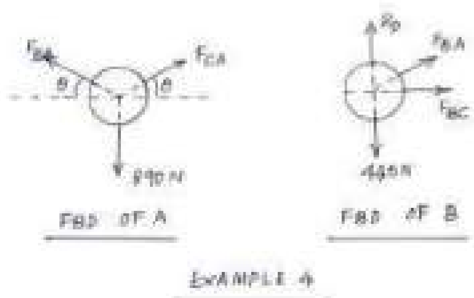
$$\frac{120}{Sin90^\circ}=\frac{F}{Sin(180-32)^\circ}=\frac{NR}{Sin(90+32)^\circ}$$

$F = 63.59N$

$N_R = 101.76N$

Example 4: Three smooth circular cylinders are placed in an arrangement as shown. Two cylinders are of radius 052mm and weight 445 N are kept on a horizontal surface. The centers of these cylinders are tied by a string which is 406 mm long. On these two cylinders, third cylinder of weight 890N and of same diameter is kept. Find the force S in the string and also forces at points of contact.

- LAMI'S Theorem



FBD of A

$F_{AC} = 598N$

$F_{BA} = 598\ N$

FBD of B

$\sum f_{x_i} = 0$

$\sum f_{y_i} = 0$

$F_{BC} = 399.5N$

$R_D = 890N$

1.A 200 N sphere is resting in at rough as shown in fig. determine the reactions developed at contact surfaces. Assume all contact surfaces are smooth.

Soln. At contact point 1, the surface contact is making 60^0 to horizontal. Hence the reaction R_1 which is normal to it makes 60^0 with vertical. Similarly the reaction R_2 at contact point 2 makes 45^0 to the vertical. FBD as shown in figure.

Applying lami's theorem to the system of forces, we get

$R_1/\sin(180-45) = R_2/\sin(180-60) = 400/\sin(60+45)$

$R_1 = 292.8N$ $R_2 = 358.6N$

A wire is fixed at A and D as shown in figure. Weights 20 kN and 25kN are supported at B and C respectively. When equilibrium is reached it is found that inclination o fAB is 300 and that of CD I s600 to the vertical. Determine the tension in the segments AB, BC, and CD of the rope and also the inclination of BC to the vertical.

Soln; Free body diagrams of the point B and C are shown in figures respectively.

Considering equilibrium o fpoint B, we get

$\sum Fx = 0$

$T_2 \sin\theta - T_1 \sin 30 = 0$

$T_2 \sin\theta = T_1 \sin 30$ --------------(i)

$\sum Fy = 0$

$-T_2 \cos\theta + T_1 \cos 30 - 20 = 0$

$T_2 \cos\theta = T_1 \cos 30 - 20$ --------------(ii)

Considering the equilibrium of point C,

$\sum Fx = 0$

$T_3 \sin 60 - T_2 \sin\theta = 0$

$T_2 \sin\theta = T_3 \sin 60$ --------------(iii)

$\sum Fy = 0$

$T_3 \cos 60 + T_2 \cos\theta - 25 = 0$

$T_2 \cos\theta = -T_3 \cos 60 + 25$ --------------(iv)

From equations (i) and (ii) , we get

$T_1 \sin 30 = T_3 \sin 60$

$T_1 = \sqrt{3}T_3$

From equations (ii) and (iv) , we get

$T_1 \cos 30 - 20 = -T_3 \cos 60 + 25$

$T_3 = 22.5kN$

$T_1 = 38.97kN$

From equation (i) and(ii)

$\tan\theta = 1.416$

$\theta = 54.78^o$

$T_2 = 23.84kN$

A ladder weighing 100N is to be kept in the position shown in figure. Resting on a smooth floor and leaning on a smooth wall. Determine the horizontal force required at floor level to prevent it from slipping when a man weighing 700 N is at 2 m above floor level.

Free body diagram of the ladder is as shown in figure. Ra is vertical and Rb is horizontal because the surface of contact is smooth. Self weight of 100N acts through centre point of ladder in vertical direction. Let F be the horizontal force required to be applied to prevent slipping.

Then

$\sum M_A = 0$

$-R_B \times 3 + 700 \times 2 \cot 60 + 100 \times 1.5 \cot 60 = 0$

$R_B = 298.3N$

$\sum Fx = 0$

$F - R_B = 0$

$F - R_B = 298.3N$

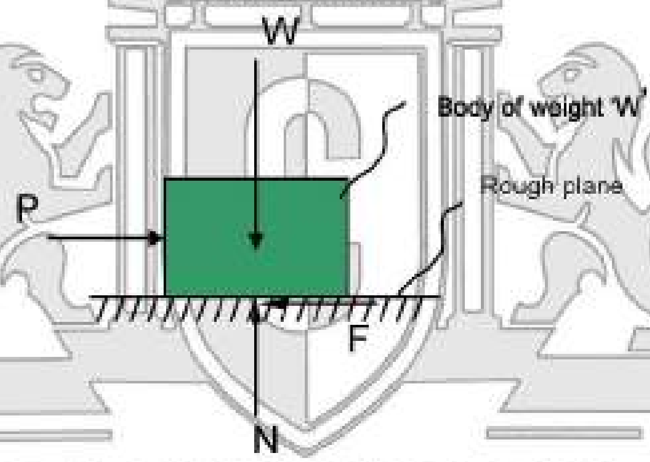
FRICTION

Whenever a body moves or tends to move over another surface or body, a force which opposes the motion of the body is developed tangentially at the surface of contact, such on opposing force developed is called friction or frictional resistance.

The frictional resistance is developed due to the interlocking of the surface irregularities at the contact surface b/w two bodies.

Consider a body weighing W resting on a rough plane & subjected to a force 'P'

to displace the body.

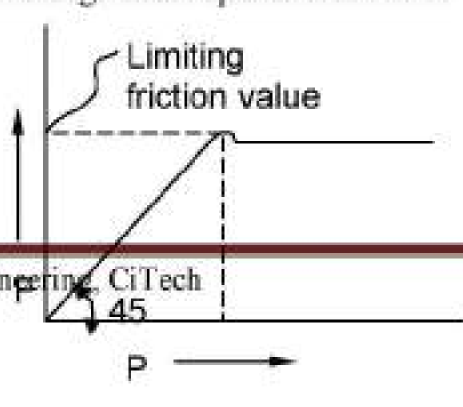


Where

P = Applied force
N = Normal reaction from rough surface
F = Frictional resistance
W = Weight of the body

The body can start moving or slide over the plane if the force 'P' overcomes the frictional 'F'

The frictional resistance developed is proportional to the magnitude of the applied force which is responsible for causing motion upto a certain limit.



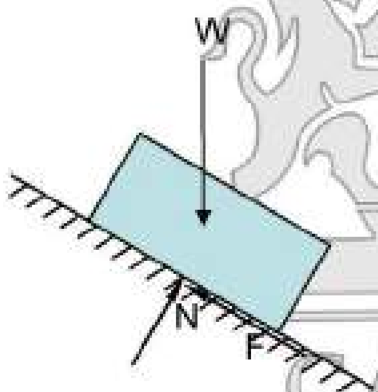
From the above graph we see that as P increases, F also increases. However F cannot increase beyond a certain limit. Beyond this limit (Limiting friction value) the frictional resistance becomes constant for any value of applied force. If the magnitude of the applied force is less than the limiting friction value, the body remains at rest or in equilibrium. If the magnitude of the applied force is greater than the limiting friction value the body starts moving over the surface.

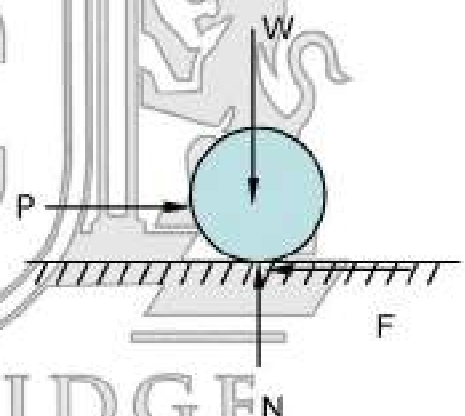
The friction experienced by a body when it is at rest or in equilibrium is known as static friction. It can range between a zero to limiting fraction value.

The friction experienced by a body when it is moving is called **dynamic friction**.

The dynamic friction experienced by a body as it slides over a plane as it is shown in figure is called **sliding friction.**

The dynamic friction experienced by a body as it roles over surface as shown in figure is called **rolling friction**.





CO-EFFICIENT OF FRICTION: It ha been experimentally proved that between two contacting surfaces, the magnitude of limiting friction bears a constant ratio to normal reaction between the two this ratio is called as co-efficient of friction.

It is defined by the relationship $\mu = \frac{F}{N}$

Where

μ – Represents co-efficient of friction
F – Represents frictional resistance
N – Represents normal reaction.

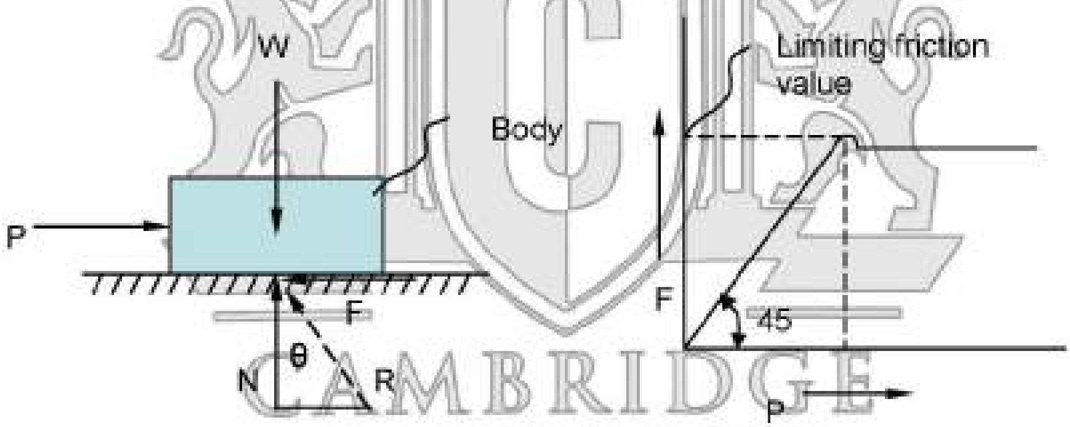
Note: Depending upon the nature of the surface of contact i.e. dry surface & wet surface, the frictional resistance developed at such surface can be called dry friction & wet friction (fluid friction) respectively. In our discussion on friction all the surface we consider will be dry sough surfaces.

LAWS OF DRY FRICTION: (COLUMB'S LAWS)

The frictional resistance developed between bodies having dry surfaces of contact obey certain laws called laws of dry friction. They are as follows.

1) The frictional resistance depends upon the roughness or smoothness of the surface.
2) Frictional resistance acts in a direction opposite to the motion of the body.
3) The frictional resistance is independent of the area of contact between the two bodies.
4) The ratio of the limiting friction value (F) to the normal reaction (N) is a constant (co-efficient of friction, μ)
5) The magnitude of the frictional resistance developed is exactly equal to the applied force till limiting friction value is reached or where the bodies is about to move.

ANGLE OF FRICTION



Consider a body weighing 'W' placed on a horizontal plane. Let 'P' be an applied force required to just move the body such that, frictional resistance reaches limiting friction value. Let 'R' be resultant of F & N. Let 'θ' be the angle made by the resultant with the direction of N. such an angle 'θ' is called the Angle of friction

As P increases, F also increases and correspondingly 'θ' increases. However, F cannot increase beyond the limiting friction value and as such 'θ' can attain a maximum value only.

Let $\theta_{max} = \alpha$

Where α represents angle of limiting friction

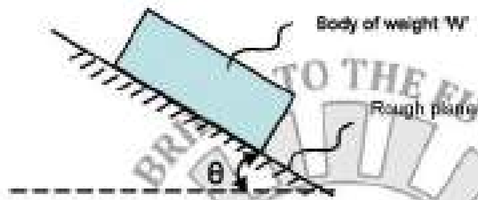
$$\tan \theta_{max} = \tan\alpha = \frac{F}{N}$$

But $\frac{F}{N} = \mu$

Therefore $\mu = \tan\alpha$

i.e. co-efficient of friction is equal to the tangent of the angle of limiting friction

ANGLE OF REPOSE:



Consider a body weighing 'w' placed on a rough inclined plane, which makes an angle 'θ' with the horizontal. When 'θ' value is small, the body is in equilibrium or rest without sliding. If 'θ' is gradually increased, a stage reaches when the body tends to slide down the plane

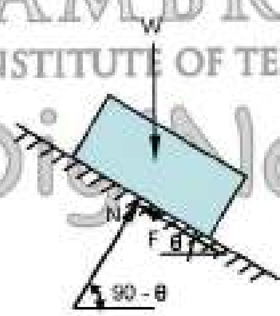
The maximum inclination of the plane with the horizontal, on which a body free from external forces can rest without sliding is called angle of repose.

Let $\theta_{max} = \Phi$

Where Φ = angle of repose

When = angle of repose.

Let us draw the free body diagram of the body before it slide.



Applying conditions of equilibrium.

$$\sum F_x = 0$$

$$N\cos(90-\theta) - F\cos\theta = 0$$

$$N\sin\theta = F\cos\theta$$

$$\tan\theta = \frac{F}{N}$$

$$\tan\theta_{max} = \tan\Phi$$

$$\text{but } \frac{F}{N} = \mu$$

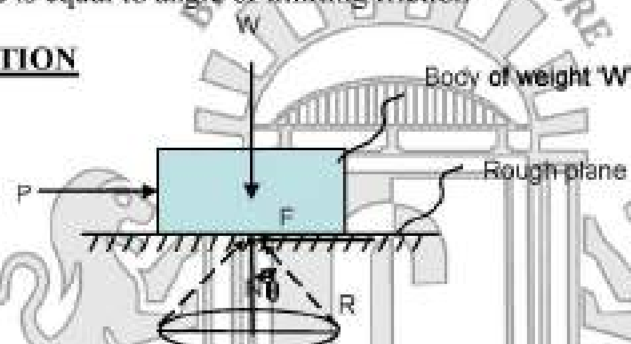
$$\mu = \tan\alpha$$

$$\tan\Phi = \tan\alpha$$

$$\Phi = \alpha$$

i.e. angle of repose is equal to angle of limiting friction

CONE OF FRICTION



Consider a body weighting 'W' resting on a rough horizontal surface. Let 'P' be a force required to just move the body such that frictional resistance reaches limiting value. Let 'R' be the resultant of 'F' & 'N' making an angel with the direction of N.

If the direction of 'P' is changed the direction of 'F' changes and accordingly 'R' also changes its direction. If 'P' is rotated through 360°, R also rotates through 360° and generates an imaginary cone called cone of friction.

Note: In this discussion, all the surface that bee consider are rough surfaces, such that, when the body tends to move frictional resistance opposing the motion comes into picture tangentially at the surface of contact in all the examples, the body considered is at the verge of moving such that frictional resistance reaches limiting value. We can consider the body to be at rest or in equilibrium & we can still apply conditions of equilibrium on the body to calculate unknown force.

Ex. Block A weighing 1000 N rests over block B which weighs 2000 N as shown in Fig. Block A is tied to wall with a horizontal string. If the coefficient of friction between A and B is 1/4 and between B and the floor is 1/3, what should be the value of P to move the block B if (a) P is horizontal? (b) P acts 30° upwards to horizontal?

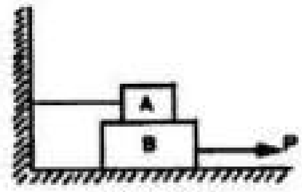
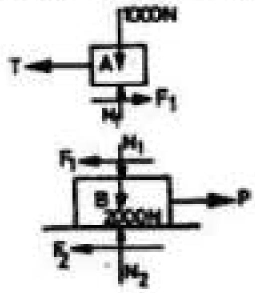


Fig. 5.5(a)



(a) *When P is horizontal:*

The free body diagrams of the two blocks are shown in Fig. Note the frictional forces F_1 and F_2 are to be marked in the opposite direction of impending relative motion. Considering block A,

$$\Sigma V = 0$$

$$N_1 = 1000\text{ N}$$

since F_1 is limiting friction, $\dfrac{F_1}{N_1} = \dfrac{1}{4}$

$$\therefore \quad F_1 = 250\text{ N}$$

$$\Sigma H = 0$$

$$T = F_1$$

$$= 250\text{ N}$$

Considering block B,

$$\Sigma V = 0$$

$$N_2 - 2000 - N_1 = 0$$

$$N_2 = 3000\text{ N} \quad \text{since } N_1 = 1000\text{ N}$$

Since F_2 is the limiting friction $F_2 = \mu_2 N_2$

$$= \frac{1}{3} \times 3000 = 1000 \text{ N}$$

$$\Sigma H = 0$$

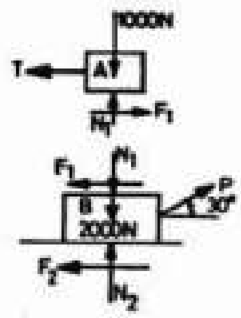
$$P - F_1 - F_2 = 0$$

$$P = F_1 + F_2 = 250 + 1000$$

$$= 1250 \text{ N}$$

(b) *When P is enclined*

Free body diagram for this case is shown in Fig.



As in the previous case, here also N_1 = 1000 N and F_1 = 250 N. Consider the equilibrium of block *B*.

$$\Sigma V = 0$$

$$N_2 - 2000 - N_1 + P \sin 30° = 0$$

$$N_2 + 0.5\,P = 3000 \text{ N} \qquad \text{since } N_1 = 1000 \text{ N}$$

From law of friction,

$$F_2 = \frac{1}{3} N_2$$

$$= \frac{1}{3}(3000 - 0.5\,P)$$

$$= 1000 - \frac{0.5}{3} P$$

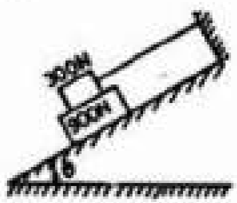
$\Sigma H = 0$

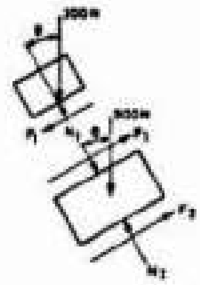
$$P \cos 30^\circ - F_1 - F_2 = 0$$

$$P \cos 30^0 - 250 - (1000 - \frac{0.5}{3} P) = 0$$

$\therefore$ **P = 1210.43 N** *Ans.*

Ex. What should be the value of θ in Fig. which will make the motion of 900 N block down the plane to impend? The coefficient of friction for all contact surfaces is $\frac{1}{3}$.





900 N block is on the verge of moving downward. Hence frictional forces F_1 and F_2 act up the plane on 900 N block. Free body diagram of the blocks is as shown in Fig.

For 300 N block:

Σ forces normal to plane = 0

$N_1 - 300 \cos\theta = 0$

or $N_1 = 300 \cos\theta$ ...(1)

From law of friction $F_1 = \frac{1}{3} N_1 = 100 \cos\theta$ ...(2)

For 900 N block:

Σ forces normal to the plane = 0

$N_2 - N_1 - 900 \cos\theta = 0$

or $N_2 = N_1 + 900 \cos\theta$

Substituting the value of N_1 from (1) we get

$N_2 = 1200 \cos\theta$ ...(3)

From law of friction

$F_2 = \frac{1}{3} N_2 = 400 \cos\theta$ ...(4)

Σ forces parallel to the plane = 0

$$F_1 + F_2 - 900 \sin\theta = 0$$

i.e. $100 \cos\theta + 400 \cos\theta = 900 \sin\theta$

$$\tan\theta = \frac{5}{9}$$

$\therefore \quad \theta = 29.05°$ *Ans.*

Ex. A weight 500 N just starts moving down a rough inclined plane supported by a force of 200 N acting parallel to the plane and it is at the point of moving up the plane when pulled by a force of 300 N parallel to the plane. Find the inclination of the plane and the coefficient of friction between the inclined plane and the weight.

Free body diagram of the block when it just starts moving down is shown in Fig. and when just starts moving up is shown in the Fig. Now, frictional forces oppose the direction of the movement of the block and since it is limiting case

$$\frac{F}{N} = \mu$$

When the block just starts moving down

Σ forces perpendicular to the plane = 0 ...(1)

$$N = 500 \cos\theta$$

From law of friction, $F_1 = \mu N$

i.e. $F_1 = 500\,\mu \cos\theta$ ...(2)

$\therefore$ Σ forces parallel to the plane = 0

$$500 \sin\theta - F_1 - 200 = 0$$

i.e $500 \sin\theta - 500\ \mu\ \cos\theta = 200$ ...(3)

When the block just starts moving up the plane

Σ forces perpendicular to the plane = 0

$$N = 500 \cos\theta \quad ...(4)$$

From the law of friction, $F_2 = 500\,\mu \cos\theta$ ...(5)

Σ forces parallel to the plane = 0

$$500 \sin\theta + F_2 - 300 = 0$$

i.e $$500 \sin\theta + 500\,\mu \cos\theta = 300 \quad ...(6)$$

Adding (3) and (6) we get

$$1000 \sin\theta = 500$$

$$\sin\theta = \frac{1}{2}$$

or $\theta = 30°$ *Ans.*

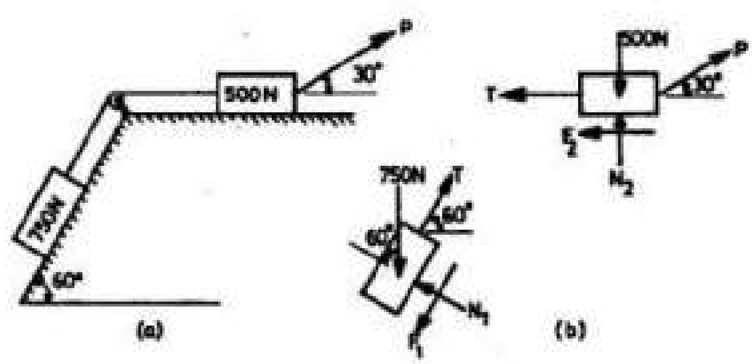
Substituting it in Eqn. (6) we get:

$$500\,\mu \cos 30° = 300 - 500 \sin 30°$$

$$= 50$$

$\therefore \quad \mu = 0.11547$ *Ans.*

Ex. What is the value of *P* in the system shown in Fig. to cause the motion to impend? Assume the pulley is smooth and coefficient of friction between the other contact surfaces is 0.20.



Free body diagrams of the blocks are as shown in Fig. Considering 750 N block:

Σ forces normal to the plane = 0

$N_1 - 750 \cos 60° = 0$

$N_1 = 375$ N

Since the motion is impending, from law of friction,

$F_1 = \mu N_1 = 0.2 \times 375$

$= 75$ N

Σ forces parallel to the plane = 0

$T - F_1 - 750 \sin 60° = 0$

$T = 75 + 750 \sin 60°$

$= 724.52$ N

Considering 500 N body:

$\Sigma V = 0$

$N_2 - 500 + P \sin 30° = 0$

$N_2 + 0.5 P = 500$

From law of friction,

$F_2 = 0.2 N_2$

$= 0.2 (500 - 0.5 P)$

$= 100 - 0.1 P$

$\Sigma H = 0$

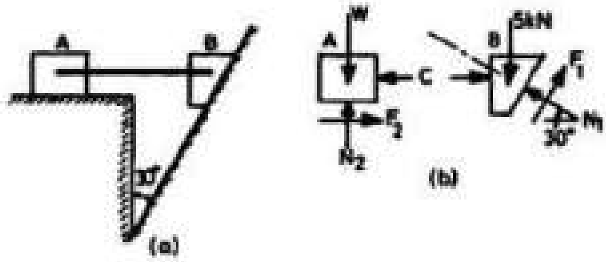
$P \cos 30° - T - F_2 = 0$

$P \cos 30° - 724.52 - 100 + 0.1 P = 0$

P = 853.52 N *Ans.*

Ex. Two blocks connected by a horizontal link *AB* are supported on two rough planes as shown in Fig. The coefficient of friction for the block on the horizontal plane is 0.4. The limiting angle of friction for block *B* on the inclined plane is 20°. What is the smallest weight *W* of the block *A* for which equilibrium of the system can exist if weight of block *B* is 5 kN?

Free body diagrams for block *A* and *B* are as shown in Fig.
Consider block *B*.
From law of friction,



$$F_1 = N_1 \tan 20° \qquad \text{[Since } \mu = \tan 20°]$$

$$\Sigma V = 0$$

$$N_1 \sin 30° + F_1 \sin 60° - 5 = 0$$

$$0.5N_1 + N_1 \tan 20° \sin 60° = 5$$

$$\therefore \quad N_1 = 6.133 \text{ kN}$$

$$\therefore \quad F_1 = 6.133 \tan 20° = 2.232 \text{ kN}$$

$$\Sigma H = 0$$

$$C + F_1 \cos 60° - N_1 \cos 30° = 0$$

$$C = 6.133 \cos 30° - 2.232 \cos 60°$$

$$= 4.196 \text{ kN}$$

Now consider the equilibrium of block *A*.

$$\Sigma H = 0$$

$$F_2 = C = 4.196 \text{ kN}$$

$$F_2 = C = 4.196 \text{ kN}$$

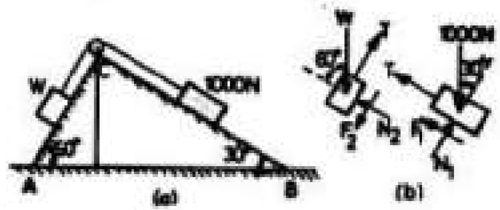
From law of friction $F_2 = \mu N_2$

i.e $N_2 = \frac{4.196}{0.4} = 10.49$ kN

$\Sigma V = 0$

W = N2 = 10.49 kN *Ans.*

Ex. Two identical planes *AC* and *BC* inclined at 60° and 30° to the horizontal meet at *C*. A load of 1000 N rests on the inclined plane *BC* and is tied by a rope passing over a pulley to a block weighing *W* Newtons and resting on the plane *AC* as shown in Fig. If the coefficient of friction between the load and the plane *BC* is 0.28 and that between the block and the plane *AC* is 0.20, find the least and the greatest value of *W* for the equilibrium of the system.



For the least value of *W* for equilibrium, the motion of 1000 N block is impending downward. For such a case the free body diagram of blocks are shown in Fig.
Considering the 1000 N block:

Σ forces normal to plane = 0

$N_1 = 1000 \cos 30° = 866.03$ N

From the law of friction $F_1 = 0.28\, N_1$

$= 242.49$ N

Σ forces parallel to the plane = 0

$T = -F_1 + 1000 \sin 30°$

$= 257.51$ N

Now consider the equilibrium of block of weight *W*:

Σ Forces normal to the plane = 0

$N_2 = W \cos 60° = 0.5\, W$

$\therefore \quad F_2 = 0.2\, N_2 = 0.1\, W$

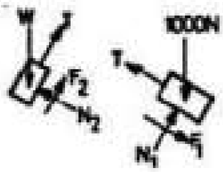
Σ forces parallel to the plane = 0

$$F_2 + W \sin 60° = T$$

$$0.1\, W + W \sin 60° = 257.51$$

$$\therefore\ \mathbf{W = 266.57\ N}$$ *Ans.*

For the greatest value of W, the 1000 N block is on the verge of moving up the plane. For such a case, the free body diagrams of the blocks are as shown in Fig.



Considering block of 1000 N,

$$N_1 = 866.03\ N$$

$$F_1 = 242.49\ N$$

$$T = 1000 \sin 30° + F_1 = 742.49\ N$$

Considering block of weight W,

$$N_2 = W \cos 60° = 0.5\, W$$

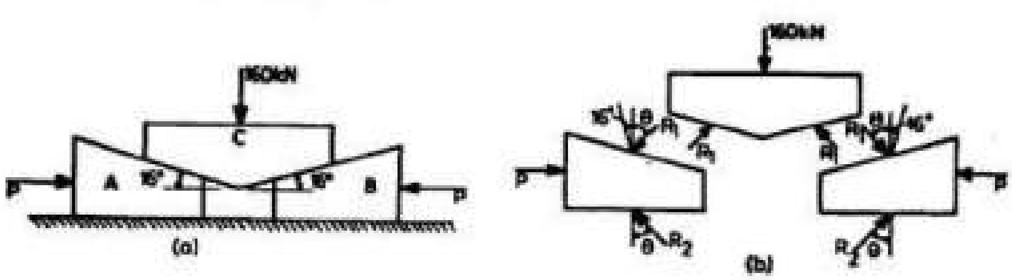
$$F_2 = 0.2\, N_2 = 0.1\, W$$

and $W \sin 60° - F_2 = T$

$$W(\sin 60° - 0.1) = 742.49$$

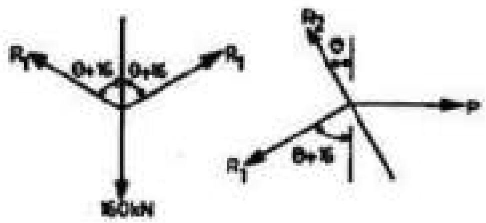
$$\mathbf{W = 969.28\ N}$$ *Ans.*

Ex. A weight of 160 kN is to be raised by means of the widges A and B as shown in Fig. Find the value of force P for impending motion of block C upwards, if coefficient of friction is 0.25 for all surfaces. Weights of the block C and the wedges may be neglected.



Let α be angle of limiting friction. Then,

$$\theta = \tan^{-1}(0.25) = 14.036°$$



The free body diagrams of A, B and C are as shown in Fig. The problem being symmetric, the forces R_1 and R_2 on wedges A and B are the same. The system of forces on block C and on wedge A are shown in the form convenient for applying Lami's theorem

Consider the equilibrium of block C.

$$\frac{R_1}{\sin(180 - 16 - \theta)} = \frac{160}{\sin 2(\theta + 16)}$$

i.e. $$\frac{R_1}{\sin 149.96} = \frac{160}{\sin 60.072°}$$

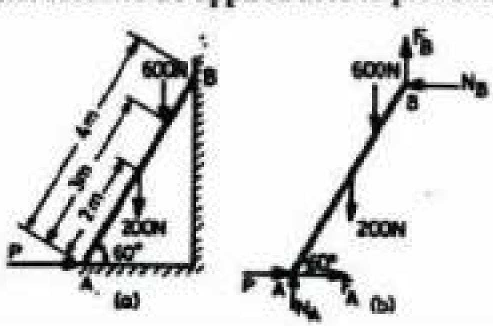
$$R_1 = 92.41 \text{ kN}$$

Consider the equilibrium of wedge A:

$$\frac{P}{\sin(180 - \theta - \theta - 16)} = \frac{R_1}{\sin(90 + \theta)}$$

P = 66.256 kN *Ans.*

Ex. A ladder of length 4 m weighing 200 N is placed against a vertical wall as shown in Fig. The coefficient of friction between the wall and the ladder is 0.2 and that between the floor and the ladder is 0.3. The ladder in addition to its own weight has to support a man weighing 600 N at a distance of 3 m from A. Calculate the minimum horizontal force to be applied at A to prevent slipping.



The free body diagram of the ladder is as shown in Fig.

$$\Sigma M_A = 0$$

$$N_B\, 4 \sin 60° + F_B\, 4 \cos 60° - 600 \times 3 \cos 60° - 200 \times 2 \cos 60° = 0$$

Dividing throughout by 4 and rearranging,

$$N_B\, 0.866 + 0.5\, F_B = 275$$

From the law of friction, $F_B = 0.2\, N_B$

$$\therefore \quad N_B\,(0.866 + 0.5 \times 0.2) = 275$$

$$N_B = 284.68 \text{ N}$$

$$\therefore \quad F_B = 56.934 \text{ N}$$

$$\Sigma V = 0$$

$$N_A - 200 - 600 + 56.934 = 0$$

$$N_A = 743.066 \text{ N}$$

$$\therefore F_A = 0.3\, N_A$$

$$\therefore F_A = 222.92 \text{ N}$$

$$\Sigma H = 0$$

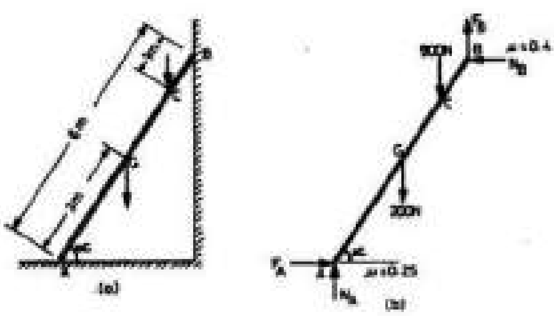
$$P + F_A - N_B = 0$$

$$P = N_B - F_A = 284.68 - 222.92$$

$$P = 61.76 \text{ N}$$

Ans.

Ex. The ladder shown in Fig. is 6 m long and is supported by a horizontal floor and vertical wall. The coefficient of friction between the floor and the ladder is 0.25 and between wall and ladder is 0.4. The weight of ladder is 200 N and may be considered as concentrated at *G*. The ladder also supports a vertical load of 900 N at *C* which is at a distance of 1m from *B*. Determine the least value of α at which the ladder may be placed without slipping. Determine the reaction at that stage.



From the law of friction,

$$F_A = 0.25\ N_A \quad ...(1)$$

and

$$F_B = 0.4\ N_B \quad ...(2)$$

$\Sigma V = 0$

$N_A - 200 - 900 + F_B = 0$

i.e. $N_A + 0.4\ N_B = 1100$ ...(3)

$\Sigma H = 0$

$F_A - N_B = 0$

$0.25\ N_A = N_B$ ...(4)

From (3) and (4) we get:

$N_A (1 + 0.4 \times 0.25) = 1100$

$N_A = 1000\ \text{N}$ *Ans.*

$\mathbf{F_A = 250\ N}$ *Ans.*

$N_B = 250\ \text{N}$ *Ans.*

$F_B = 0.4 \times 250 = 100\ \text{N}$ *Ans.*

$\Sigma M_A = 0$

$N_B \times 6 \sin\alpha + F_B \times 6 \cos\alpha - 200 \times 3 \cos\alpha - 900 \times 5 \cos\alpha = 0$

$250 \times 6 \sin\alpha = (-100 \times 6 + 600 + 4500) \cos\alpha$

$$\tan\alpha = \frac{4500}{1500} = 3$$

$$\alpha = 71.565°$$ *Ans.*

MODULE - 3

SUPPORT REACTIONS

A beam is a structural member or element, which is in equilibrium under the action of a non-concurrent force system. The force system is developed due to the loads or forces acting on the beam and also due to the support reactions developed at the supports for the beam.

For the beam to be in equilibrium, the reactions developed at the supports the should be equal and opposite to the loads.

In a beam, one dimension (length) is considerably larger than the other two dimensions (breath & depth). The smaller dimensions are usually neglected and as such a beam is represented as a line for theoretical purposes or for analysis.

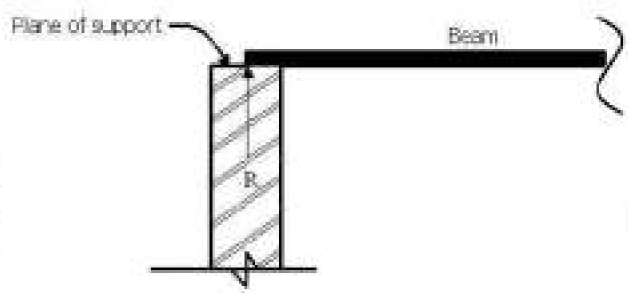
Types of Supports for beams:

Supports are structures which prevent the beam or the body from moving and help to maintain equilibrium.

A beam can have different types of supports as follows. The support reactions developed at each support are represented as follows.

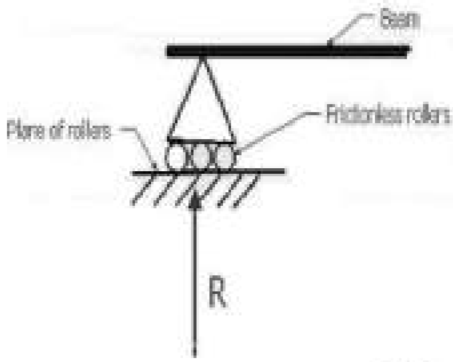
1) Simple support:

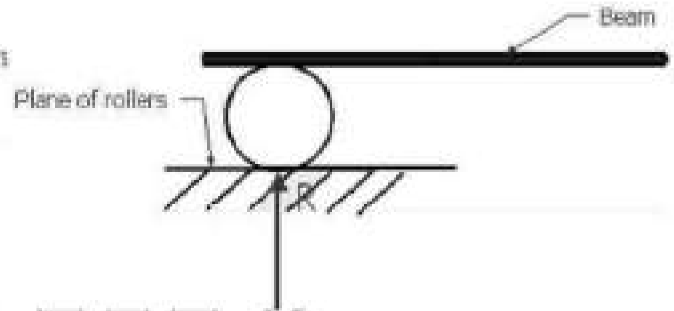
This is a support where a beam rests freely on a support. The beam is free to move only horizontally and also can rotate about the support. In such a support one reaction, which is perpendicular to the plane of support, is developed.



2) Roller support:

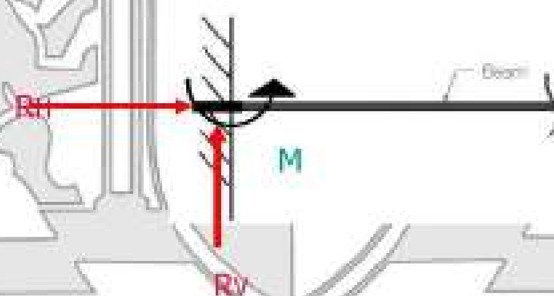
This is a support in which a beam rests on rollers, which are frictionless. At such a support, the beam is free to move horizontally and as well rotate about the support. Here one reaction which is perpendicular to the plane of rollers is developed.





4) **Fixed support**:

This is a support which prevents the beam from moving in any direction and also prevents rotation of the beam. In such a support a horizontal reaction, vertical reaction and a Fixed End Moment are developed to keep the beam in equilibrium.

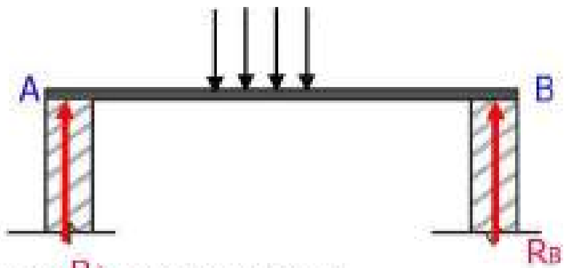


Types of beams

Depending upon the supports over which a beam can rest (at its two ends), beams can be classified as follows.

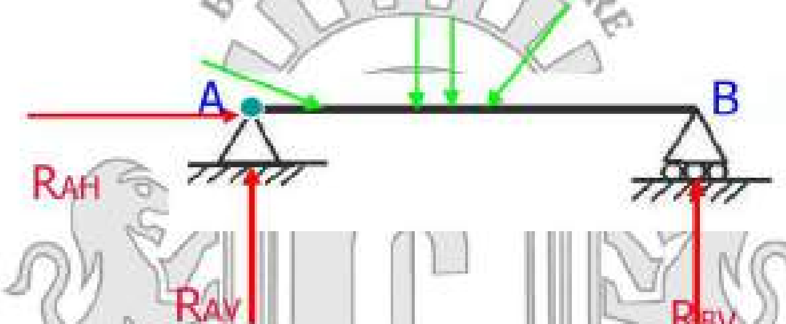
1) **Simply supported beam.**

A beam is said to be simply supported when both ends of the beam rest on simple supports. Such a beam can carry or resist vertical loads only.



2) Beam with one end hinged & other on rollers.

It is a beam where one end of the beam is hinged to a support and the other end rests on a roller support. Such a beam can carry any type of loads.

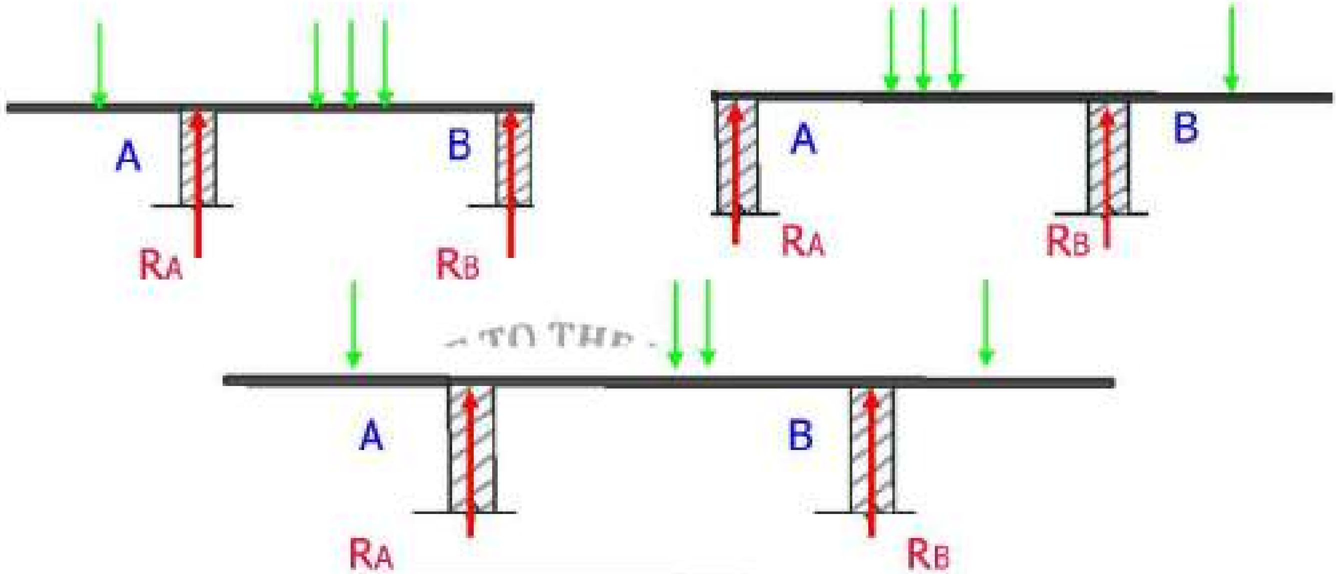


3) Hinged Beam:

It is a beam which is hinged to supports at both ends. Such a beam can carry loads is any direction.

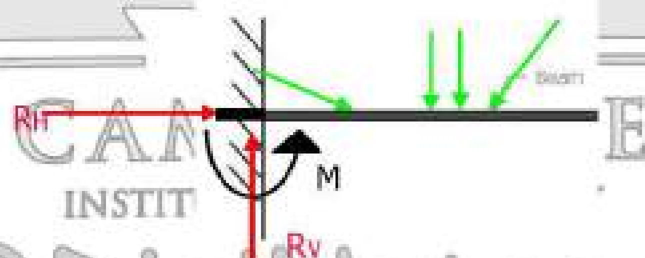
4) Over hanging beam :

It is a beam which projects beyond the supports. A beam can have over hanging portions on one side or on both sides.



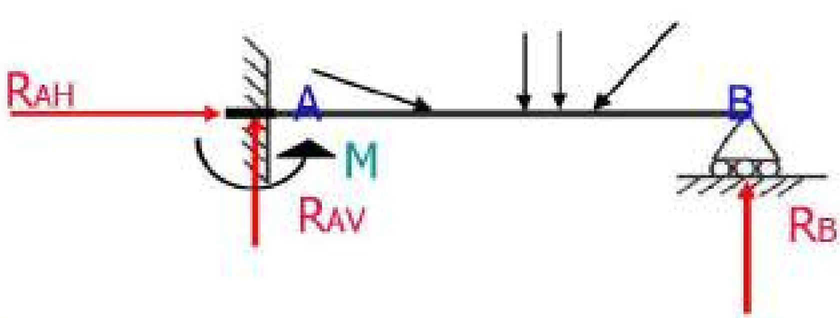
5) Cantilever Beams:

It is a beam, with one end fixed and other and free. Such a beam can carry loads in any directions.



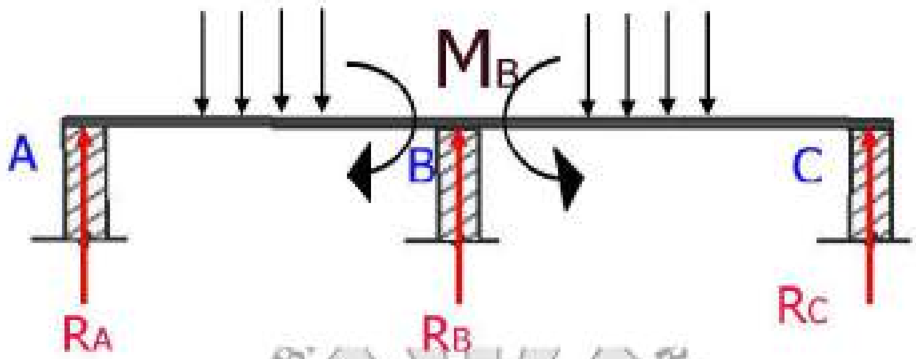
6) Propped cantilever:

It is a beam which has a fixed support at one end and a simple support at the other end.



7) Continuous beam:

It is a beam which rests over a series of supports at more than two points.



Note:

The support reactions in case of simply supported beams, beam with one end hinged and other on rollers, over hanging beams, and cantilever beams, can be determined by conditions of equilibrium only ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$). **As such, such beams are known as Statically** Determinate Beams.

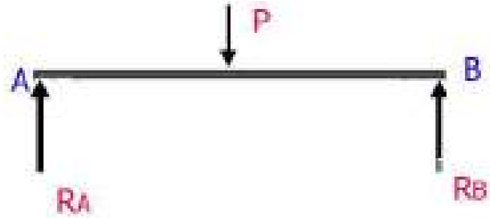
In beams such as Hinged Beams, Propped Cantilever and Continuous Beams the support reactions cannot be determined using conditions of equilibrium only. They need additional special conditions for analysis and as such, such beams are known as Statically Indeterminate Beams.

Types of loads:

The various types of loads that can act over a beam can e listed as follows.

1) Point load or Concentrated load:

If a load acts over a very small length of the beam, it is assumed to act at the mid point of the loaded length and such a loading is termed as Point load or Concentrated load.

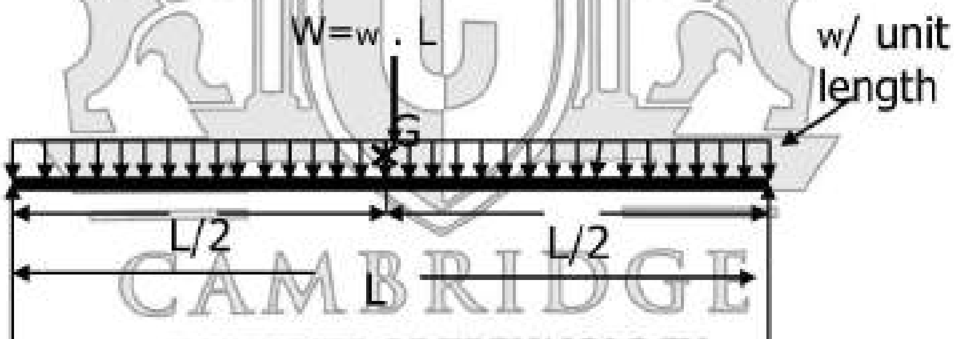


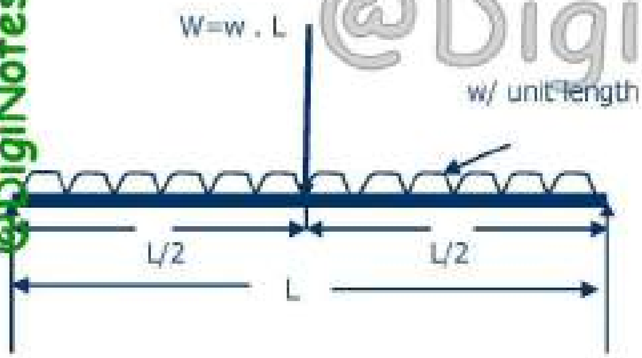
2) Uniformly distributed load (UDL):

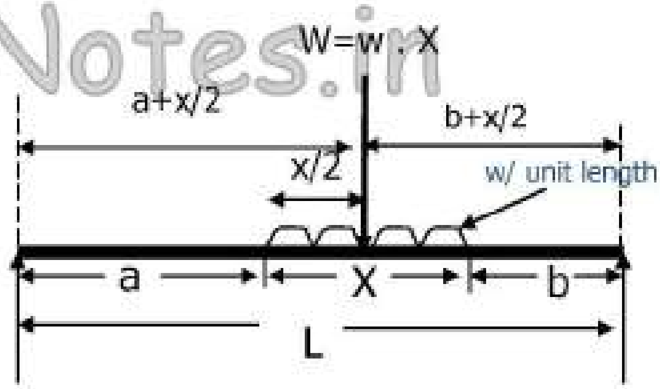
If a beam is loaded in such a manner that each unit length of the beam carries the same intensity of loading, then such a loading is called UDL.

A UDL cannot be considered in the same manner for applying conditions of equilibrium on the beam. The UDL should be replaced by an equivalent point load or total load acting through the mid point of the loaded length.

The magnitude of the point load or total load is equal to the product of the intensity of loading and the loaded length (distance).



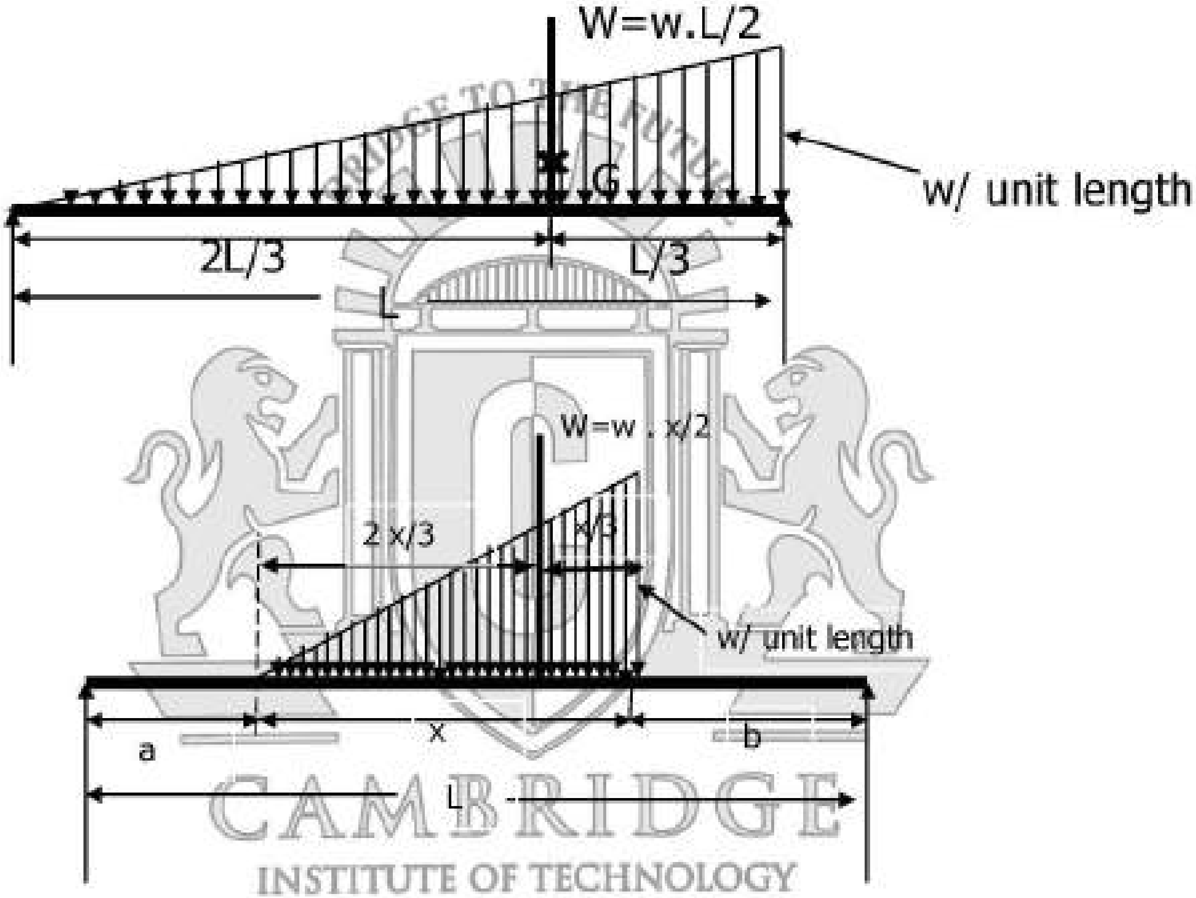




3) Uniformly varying load (UVL):

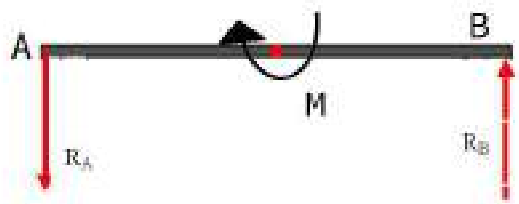
If a beam is loaded in such a manner, that the intensity of loading varies linearly or uniformly over each unit distance of the beam, then such a load is termed as UVL.

In applying conditions of equilibrium, a given UVL should be replaced by an equivalent point load or total load acting through the centroid of the loading diagram (right angle triangle). The magnitude of the equivalent point load or total load is equal to the area of the loading diagram.

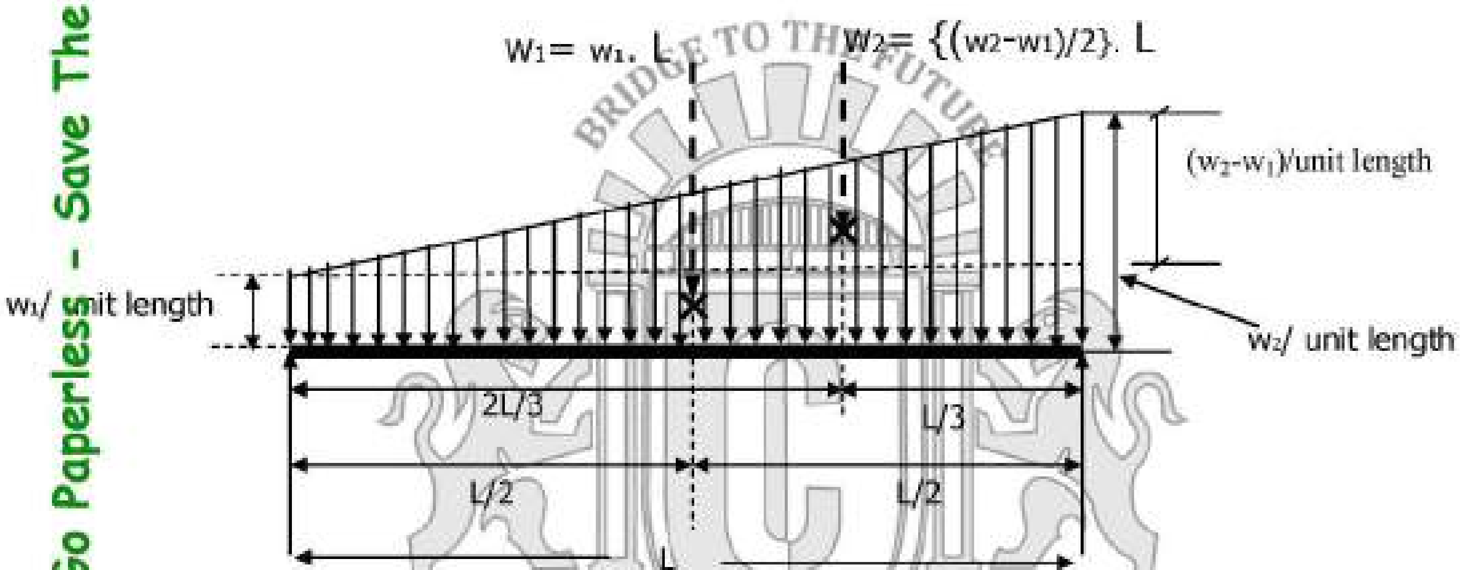


4) External moment:

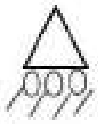
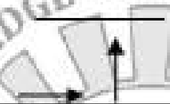
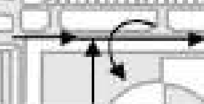
A beam can also be subjected to external moments at certain points as shown in figure. These moments should be considered while calculating the algebraic sum of moments of forces about a point on the beam



Note : A beam can also be subject to a load as shown in figure below.



In such a case, the UVL can be split into a UDL with a uniform intensity of w_1/unit length another UVL with a maximum intensity of (w_2-w_1) /unit length.

SUPPORT	REACTION	NO.OF REACTIONS
 ROLLER	V_A	(1)
HINGE		(2)
FIXED		(3)

Example 4: Determine the support reactions for the beam shown in Fig 7 at A and B.

$$\sum f_{x_i} = 0;$$
$$\sum f_{y_i} = 0;$$
$$\sum M_o = 0;$$

$$V_A - 10 - 25 - 32 + V_B = 0$$
$$V_A + V_B = 67KN;$$
$$\circlearrowleft + \sum M_A = 0$$
$$-10(2) - 25(5) - 32(9) + V_B(10) = 0$$
$$V_B = 43.3KN$$
$$V_A = 23.7KN$$

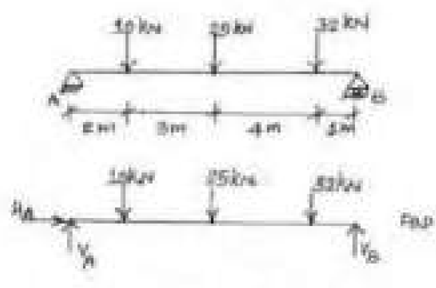


Fig-7 Example 4

Example 5: Determine the support reactions for the beam shown in Fig 8 at A and B.

$$\sum f_{x_i} = 0;\ H_A = 0$$
$$\sum f_{y_i} = 0; V_A - 40 - 40 + V_B = 0$$
$$V_A + V_B = 80$$
$$\circlearrowleft M_A = 0 - 40(2) - 40(7) + V_B(8) = 0$$
$$V_B = 45KN$$
$$V_A = 35KN;$$

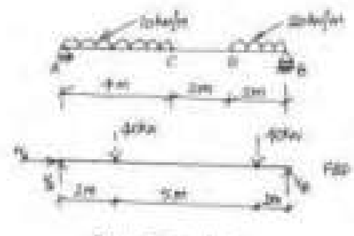


Fig 8 Example 5

Example 6: Determine the support reactions for the beam shown in Fig 9 at A and B.

$$\sum f_{x_i} = 0;$$
$$H_A - 17.32 = 0$$
$$H_A = 17.32KN$$

$$\sum f_{y_i} = 0$$
$$V_A - 10 - 20 - 15 - 10 + V_B = 0$$
$$V_A + V_B = 55$$
$$\circlearrowleft + \sum M_A = 0$$
$$-10 \times 2 + 25 - 20(6) + V_B(8) - 15(9) - 10(11) = 0$$
$$V_B = 45\text{ KN}; V_A = 10\text{ KN}$$

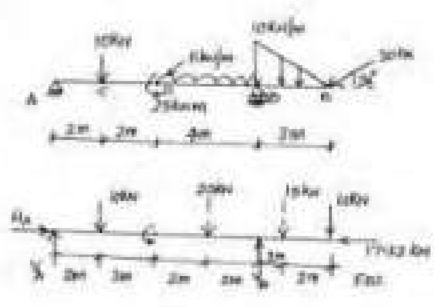


Fig 9 Example 6

Example 7: Determine the support reactions for the beam shown in Fig 10 at A and B.

$$\sum f_{x_i} = 0;$$

$$H_A - R_B \sin 30^0 = 0$$

$$H_A = 0.5 R_B$$

$$\sum f_{y_i} = 0; V_A - 20 + R_B \cos 30^0 = 0$$

$$V_A + 0.866 R_B = 20$$

$$\circlearrowleft + \sum M_B = 0;$$

$$-V_A(10) + 20(6) = 0$$

$$-V_A = 12KN;$$

$$R_B = 9.24KN;$$

$$H_A = 4.62KN;$$

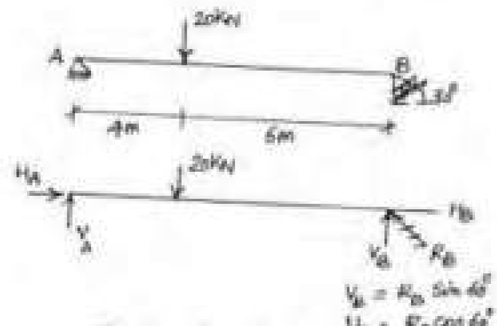


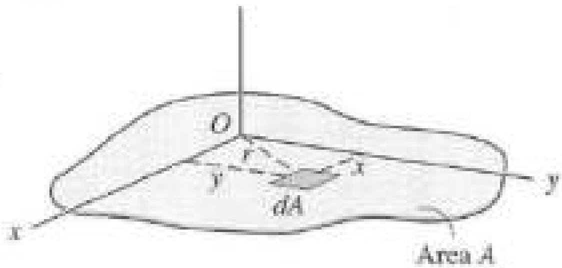
Fig 10 - Example 7

MODULE -4

MOMENT OF INERTIA

The **Moment of Inertia (I)** is a term used to describe the capacity of a cross-section to resist bending. It is always considered with respect to a reference axis such as X-X or Y-Y. It is a mathematical property of a section concerned with a surface area and how that area is distributed about the reference axis (*axis of interest*). The reference axis is usually a centroidal axis.

The moment of inertia is also known as the **Second Moment of the Area** and is expressed mathematically as:



$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$

Where

y = distance from the x axis to area *dA*
x = distance from the y axis to area *dA*

RADIUS OF GYRATION k

The **radius of gyration** of an area with respect to a particular axis is the square root of the quotient of the moment of inertia divided by the area. It is the distance at which the entire area must be assumed to be concentrated in order that the product of the area and the square of this distance will equal the moment of inertia of the actual area about the given axis. In other words, the radius of gyration describes the way in which the total cross-sectional area is distributed around its centroidal axis. If more area is distributed further from the axis, it will have greater resistance to buckling. The most efficient column section to resist buckling is a circular pipe, because it has its area distributed as far away as possible from the centroid.
Rearranging we have:

$$I_x = k_x^2 A$$
$$I_y = k_y^2 A$$

The radius of gyration is the distance *k* away from the axis that all the area can be concentrated to result in the same moment of inertia.

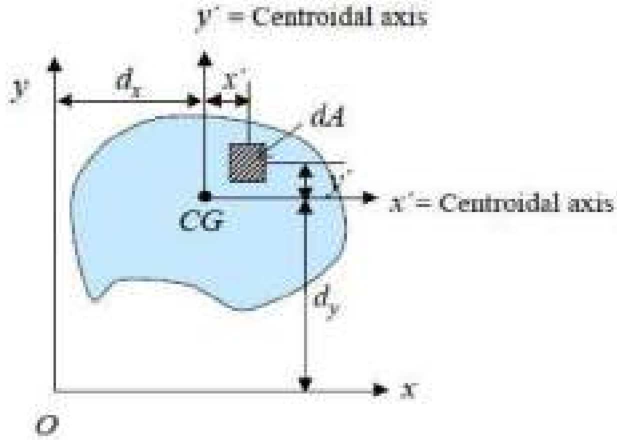
$$k_x = \sqrt{\frac{I_x}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

Parallel Axis Theorem

The moment of inertia of an area with respect to any given axis is equal to the moment of inertia with respect to the centroidal axis plus the product of the area and the square of the distance between the 2 axes.

The parallel axis theorem is used to determine the moment of inertia of composite sections.



$$I_x = \int_A (y'+d_y)^2 \, dA$$

$$= \int_A [(y')^2 + 2(y')(d_y) + (d_y)^2] \, dA$$

$$= \int_A (y')^2 \, dA + \int_A 2(y')(d_y) \, dA + \int_A (d_y)^2 \, dA$$

$0, \; \bar{y}' = 0$

$$= \bar{I}_x + 2d_y \int_A y' \, dA + d_y^2 \int_A dA$$

$$I_x = \bar{I}_x + 0 + d_y^2 A$$

$$I_y = \bar{I}_y + 0 + d_x^2 A$$

$$J_O = \bar{J}_C + Ad^2$$

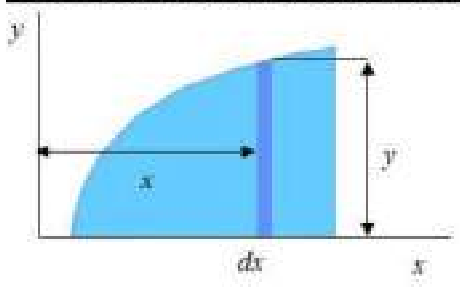
Perpendicular Axis Theorem

Theorem of the perpendicular axis states that if I_{XX} and I_{YY} be the moment of inertia of a plane section about two mutually perpendicular axis X-X and Y-Y in the plane of the section, then the moment of inertia of the section I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:

$$I_{ZZ} = I_{XX} + I_{YY}$$

The moment of inertia I_{ZZ} is also known as polar moment of inertia.

Determination of the moment of inertia of an area by integration



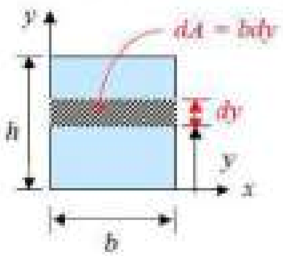
The ***rectangular moments of inertia*** I_x ***and*** I_y ***of an area*** are defined as

$$I_x = \int y^2 \, dA \qquad I_y = \int x^2 \, dA$$

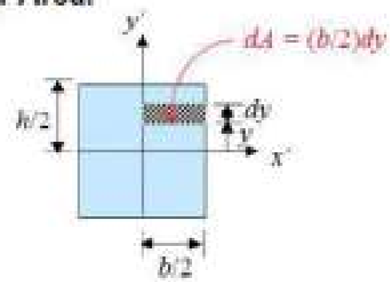
These computations are reduced to single integrations by choosing dA to be a thin strip parallel to one of the coordinate axes. The result is

$$dI_x = \frac{1}{3}y^3\,dx \qquad dI_y = x^2 y\,dx$$

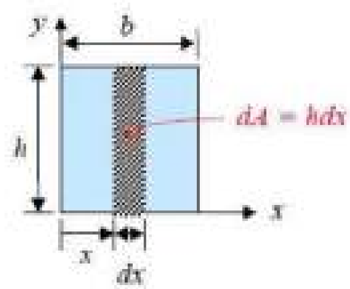
- **Moment of Inertia of a Rectangular Area.**



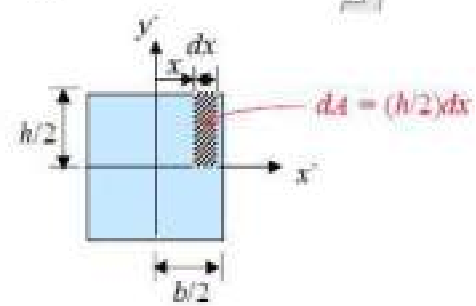
$$I_x = \int_A y^2 dA = \int_0^h y^2 (bdy) = \frac{(by^3)}{3}\Big|_0^h = \frac{bh^3}{3}$$



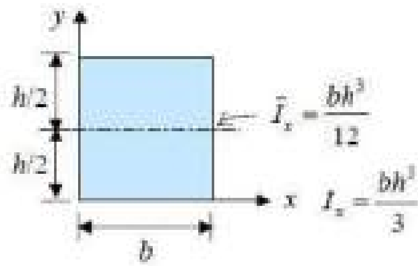
$$\bar{I}_x = I_{x'} = \int_A y^2 dA = 4\int_0^h y^2 (\frac{b}{2}dy) = 4(\frac{b}{2})\frac{y^3}{3}\Big|_0^{h/2} = \frac{bh^3}{12}$$



$$I_y = \int_A x^2 dA = \int_0^b x^2 (hdx) = \frac{(hx^3)}{3}\Big|_0^b = \frac{hb^3}{3}$$



$$\bar{I}_y = I_{y'} = \int_A x^2 dA = 4\int_0^b x^2 (\frac{h}{2}dx) = 4(\frac{h}{2})\frac{x^3}{3}\Big|_0^{b/2} = \frac{hb^3}{12}$$



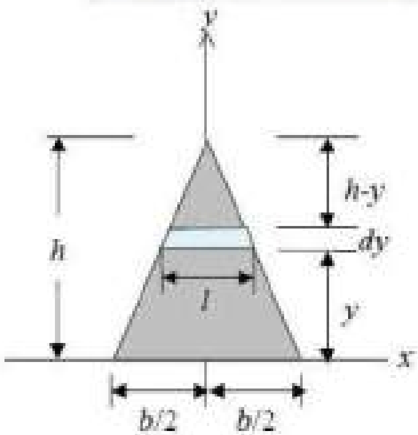
$$I_x = \bar{I}_x + Ad^2$$

$$= \frac{bh^3}{12} + (bh)\left(\frac{h}{2}\right)^2$$

$$= \frac{bh^3}{12} + \frac{bh^3}{4}$$

$$I_x = \frac{bh^3}{3}$$

• Moment of Inertia of a Triangular Area.



$dI_x = y^2\, dA \qquad dA = l\, dy$

Using similar triangles, we have

$$\frac{l}{b} = \frac{h-y}{h} \qquad l = b\frac{h-y}{h} \qquad dA = b\frac{h-y}{h}dy$$

Integrating dI_x from $y = 0$ to $y = h$, we obtain

$$I_x = \int y^2\, dA$$

$$= \int_0^h y^2 b\frac{h-y}{h}dy = \frac{b}{h}\int_0^h (hy^2 - y^3)dy$$

$$= \frac{b}{h}\left[h\frac{y^3}{3} - \frac{y^4}{4}\right]_0^h = \frac{bh^3}{12}$$

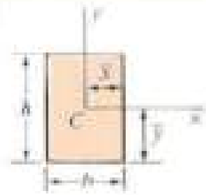
$$I_x = \bar{I}_x + Ad^2$$

$$\bar{I}_x = I_x - Ad^2$$

$$= \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36}$$

Properties of plane areas

1



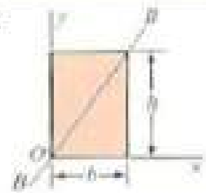
Rectangle (Origin of axes at centroid.)

$A = bh \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{2}$

$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$

$I_{xy} = 0 \qquad I_P = \frac{bh}{12}(h^2 + b^2)$

2

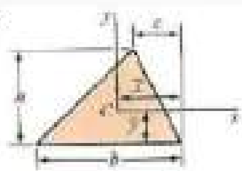


Rectangle (Origin of axes at corner.)

$I_x = \frac{bh^3}{3} \qquad I_y = \frac{hb^3}{3}$

$I_{xy} = \frac{b^2h^2}{4} \qquad I_P = \frac{bh}{3}(h^2 + b^2) \qquad I_{BB} = \frac{b^3h^3}{6(b^2 + h^2)}$

3



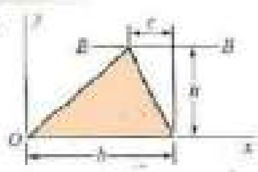
Triangle (Origin of axes at centroid.)

$A = \frac{bh}{2} \qquad \bar{x} = \frac{b + c}{3} \qquad \bar{y} = \frac{h}{3}$

$I_x = \frac{bh^3}{36} \qquad I_y = \frac{bh}{36}(b^2 - bc + c^2)$

$I_{xy} = \frac{bh^2}{72}(b - 2c) \qquad I_P = \frac{bh}{36}(h^2 + b^2 - bc + c^2)$

4

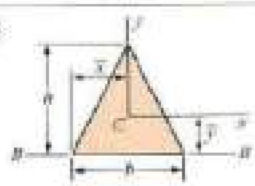


Triangle (Origin of axes at vertex.)

$I_x = \frac{bh^3}{12} \qquad I_y = \frac{bh}{12}(3b^2 - 3bc + c^2)$

$I_{xy} = \frac{bh^2}{24}(3b - 2c) \qquad I_{BB} = \frac{bh^3}{4}$

5



Isosceles triangle (Origin of axes at centroid.)

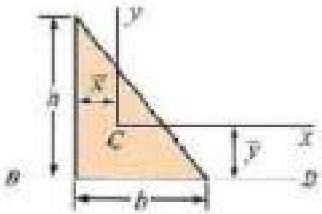
$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{3}$

$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{48} \qquad I_{xy} = 0$

$I_P = \frac{bh}{144}(4h^2 + 3b^2) \qquad I_{BB} = \frac{bh^3}{12}$

(Note: For an equilateral triangle, $h = \sqrt{3}\,b/2$.)

6



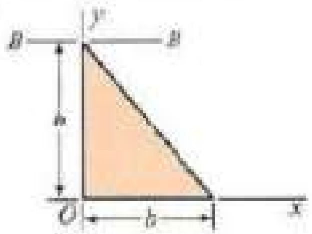
Right triangle (Origin of axes at centroid.)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{36} \qquad I_{xy} = -\frac{b^2h^2}{72}$$

$$I_P = \frac{bh}{36}(h^2 + b^2) \qquad I_{BB} = \frac{bh^3}{12}$$

7

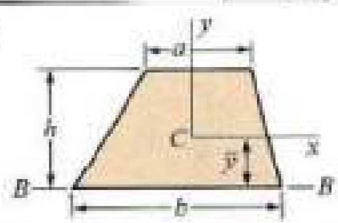


Right triangle (Origin of axes at vertex.)

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12} \qquad I_{xy} = \frac{b^2h^2}{24}$$

$$I_P = \frac{bh}{12}(h^2 + b^2) \qquad I_{BB} = \frac{bh^3}{4}$$

8

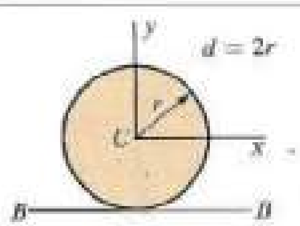


Trapezoid (Origin of axes at centroid.)

$$A = \frac{h(a + b)}{2} \qquad \bar{y} = \frac{h(2a + b)}{3(a + b)}$$

$$I_x = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)} \qquad I_{BB} = \frac{h^3(3a + b)}{12}$$

9

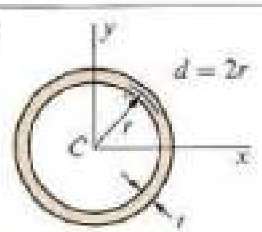


Circle (Origin of axes at center.)

$$A = \pi r^2 = \frac{\pi d^2}{4} \qquad I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

$$I_{xy} = 0 \qquad I_P = \frac{\pi r^4}{2} = \frac{\pi d^4}{32} \qquad I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$$

10

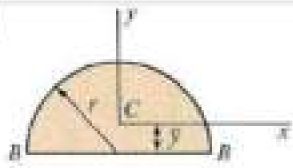


Circular ring (Origin of axes at center.)
Approximate formulas for case when t is small.

$$A = 2\pi r t = \pi d t \qquad I_x = I_y = \pi r^3 t = \frac{\pi d^3 t}{8}$$

$$I_{xy} = 0 \qquad I_P = 2\pi r^3 t = \frac{\pi d^3 t}{4}$$

11



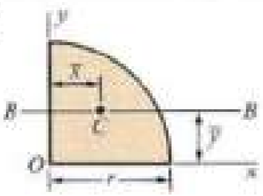
Semicircle (Origin of axes at centroid.)

$$A = \frac{\pi r^2}{2} \qquad \bar{y} = \frac{4r}{3\pi}$$

$$I_x = \frac{(9\pi^2 - 64)r^4}{72\pi} \approx 0.1098r^4 \qquad I_y = \frac{\pi r^4}{8}$$

$$I_{xy} = 0 \qquad I_{BB} = \frac{\pi r^4}{8}$$

12



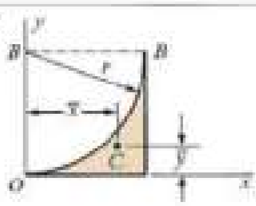
Quarter circle (Origin of axes at center of circle.)

$$A = \frac{\pi r^2}{4} \qquad \bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$I_x = I_y = \frac{\pi r^4}{16} \qquad I_{xy} = \frac{r^4}{8}$$

$$I_{BB} = \frac{(9\pi^2 - 64)r^4}{144\pi} \approx 0.05488r^4$$

13



Quarter-circular spandrel (Origin of axes at vertex.)

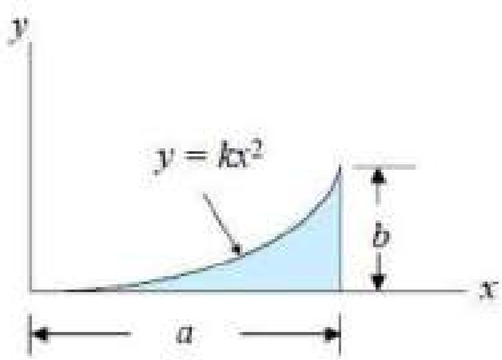
$$A = \left(1 - \frac{\pi}{4}\right)r^2$$

$$\bar{x} = \frac{2r}{3(4-\pi)} \approx 0.7766r \qquad \bar{y} = \frac{(10 - 3\pi)r}{3(4-\pi)} \approx 0.2234r$$

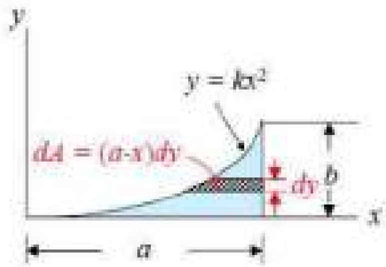
$$I_x = \left(1 - \frac{5\pi}{16}\right)r^4 \approx 0.01825r^4 \qquad I_y = I_{BB} = \left(\frac{1}{3} - \frac{\pi}{16}\right)r^4 \approx 0.1370r^4$$

Example

Determine the moment of inertia of the shaded area shown with respect to each of the coordinate axes.



- **Moment of Inertia I_x.**

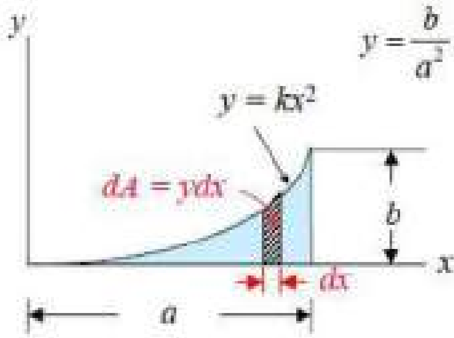


Substituting $x = a$ and $y = b$

$$y = kx^2$$
$$b = ka^2$$
$$k = \frac{b}{a^2}$$
$$y = \frac{b}{a^2}x^2 \quad or \quad x = \frac{a}{b^{1/2}}y^{1/2}$$

$$I_x = \int_A y^2 dA$$
$$= \int_0^b y^2(a - x)dy$$
$$= \int_0^b y^2\left(a - \frac{a}{b^{1/2}}y^{1/2}\right)dy$$
$$= a\int_0^b y^2 dy - \frac{a}{b^{1/2}}\int_0^b y^{5/2}dy$$
$$= \frac{ay^3}{3}\Big|_0^b - \frac{a}{b^{1/2}}\left(\frac{2}{7}y^{7/2}\right)\Big|_0^b$$
$$= \frac{ab^3}{3} - \frac{a}{b^{1/2}}\left(\frac{2}{7}b^{7/2}\right)$$
$$= \frac{ab^3}{3} - \frac{2ab^3}{7}$$
$$= \frac{ab^3}{21}$$

- **Moment of Inertia I_y.**

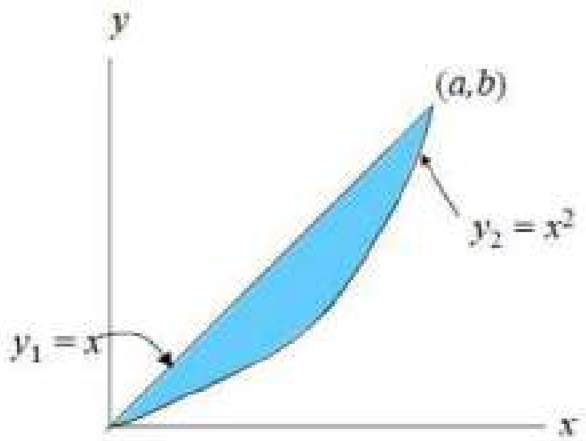


$$y = \frac{b}{a^2}x^2$$

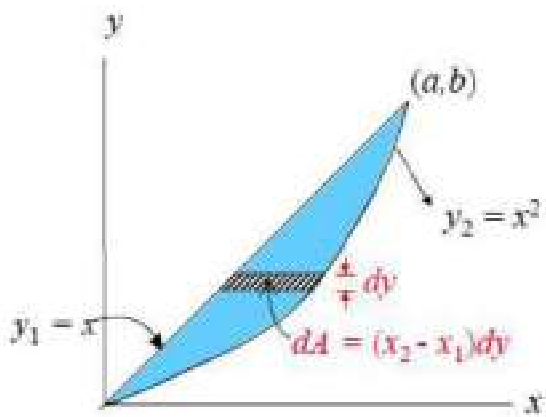
$$I_y = \int_A x^2 dA$$
$$= \int_0^a x^2 y dx$$
$$= \int_0^a x^2\left(\frac{b}{a^2}x^2\right)dx$$
$$= \frac{b}{a^2}\int_0^a x^4 dx$$
$$= \left(\frac{b}{a^2}\right)\left(\frac{x^5}{5}\right)\Big|_0^a$$
$$= \left(\frac{b}{a^2}\right)\left(\frac{a^5}{5}\right)$$
$$= \frac{a^3b}{5}$$

Example

Determine the moment of inertia of the shaded area shown with respect to each of the coordinate axes.



- **Moment of Inertia I_x.**



$$I_x = \int_A y^2 dA$$

$$= \int_0^b y^2 (x_2 - x_1) dy$$

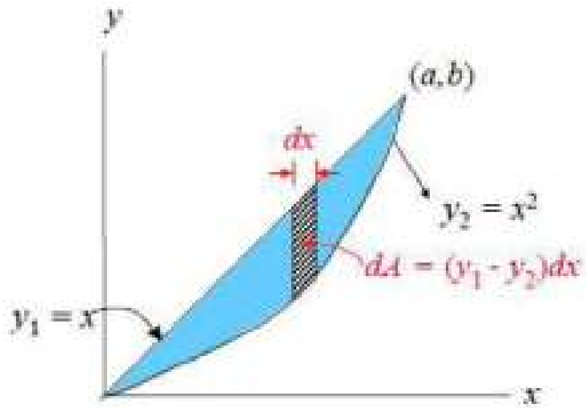
$$= \int_0^b y^2 (y^{1/2} - y) dy$$

$$= \int_0^b (y^{5/2}) dy - \int_0^b (y^3) dy$$

$$= \frac{2}{7} y^{7/2} \Big|_0^b - \frac{y^4}{4} \Big|_0^b$$

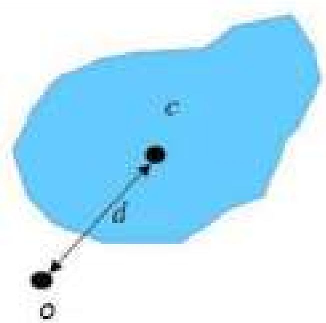
$$= \frac{2}{7} b^{7/2} - \frac{b^4}{4}$$

- **Moment of Inertia I_y.**



$$I_y = \int_A x^2 dA$$
$$= \int_0^a x^2 (y_1 - y_2) dx$$
$$= \int_0^a x^2 (x - x^2) dx$$
$$= \int_0^a (x^3) dx - \int_0^a (x^4) dx$$
$$= \left.\frac{x^4}{4}\right|_0^a - \left.\frac{x^5}{5}\right|_0^a$$
$$= \frac{a^4}{4} - \frac{a^5}{5}$$

Moment of inertia of composite areas



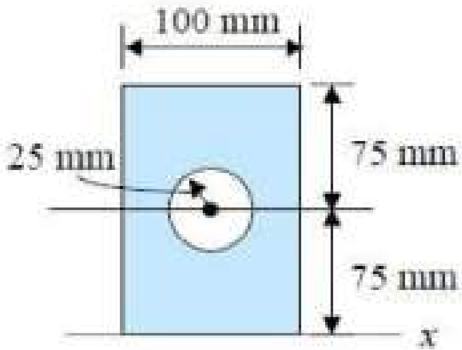
A similar theorem can be used with the polar moment of inertia. The polar moment of inertia $\overline{J}_O$ of an area about O and the polar moment of inertia J_C of the area about its centroid are related to the distance d between points C and O by the relationship

$$J_O = \overline{J}_C + Ad^2$$

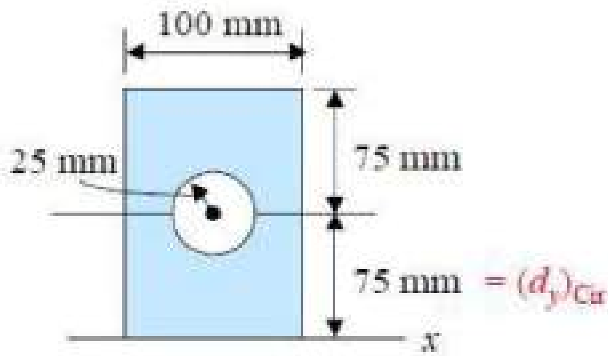
The parallel-axis theorem is used very effectively to compute the ***moment of inertia of a composite area*** with respect to a given axis.

Example

Compute the moment of inertia of the composite area shown.



SOLUTION



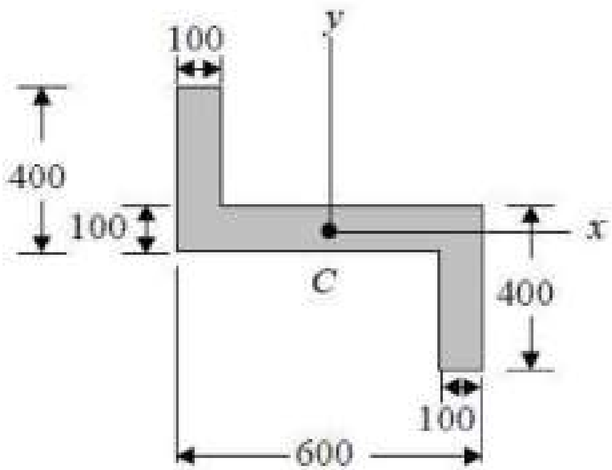
$$I_x = \left(\frac{bh^3}{3}\right)_{Rect} - (\bar{I}_x + Ad_y^{\,2})_{Cir}$$

$$= \left[\frac{1}{3}(100)(150)^3\right]_{Rect} - \left[\frac{1}{4}\pi(25)^4 + (\pi \times 25^2)(75)^2\right]_{Cir}$$

$$= 101 \text{x} 10^6 \text{ mm}^4$$

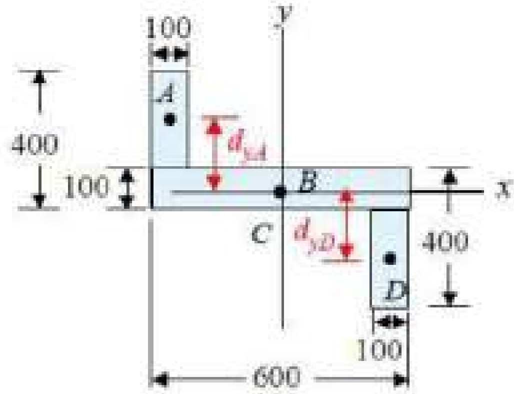
Example

Determine the moments of inertia of the beam's cross-sectional area shown about the *x* and *y* centroidal axes.



Dimension in mm

SOLUTION



Dimension in mm

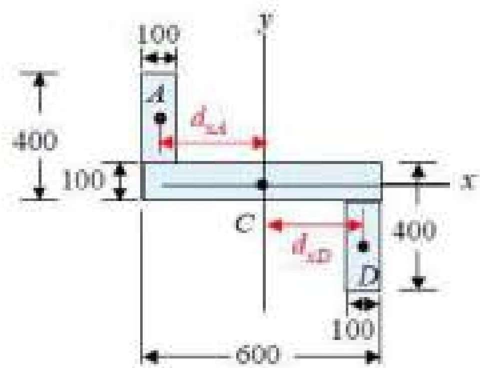
$$I_x = (\bar{I}_x + Ad_y^{\,2})_A + (\bar{I}_x + Ad_y^{\,2})_B + (\bar{I}_x + Ad_y^{\,2})_C$$

(in the printed equation, the term $Ad_y^{\,2}$ of B is struck through and marked 0)

$$= [\frac{1}{12}(100)(300)^3 + (100\times300)(200)^2] + [\frac{1}{12}(600)(100)^3 + 0]$$

$$+ [\frac{1}{12}(100)(300)^3 + (100\times300)(200)^2]$$

$$= 2.9\text{x}10^9 \text{ mm}^4$$



Dimension in mm

$$I_y = (\bar{I}_y + Ad_x^{\,2})_A + (\bar{I}_y + Ad_x^{\,2})_B + (\bar{I}_y + Ad_x^{\,2})_C$$

(in the printed equation, the term $Ad_x^{\,2}$ of B is struck through and marked 0)

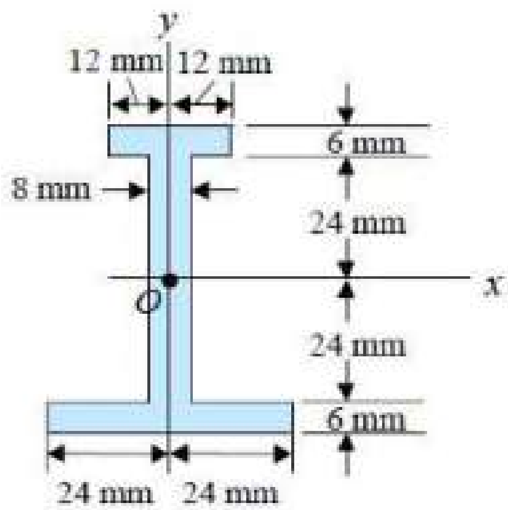
$$= [\frac{1}{12}(300)(100)^3 + (100\times300)(250)^2]_A + [\frac{1}{12}(100)(600)^3 + 0]_B$$

$$+ [\frac{1}{12}(300)(100)^3 + (100\times300)(250)^2]_C$$

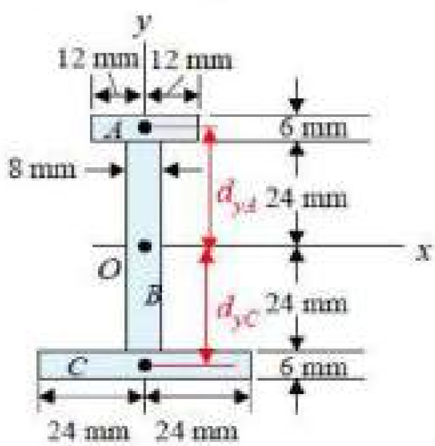
$$= 5.6\text{x}10^9 \text{ mm}^4$$

Example

Determine the moments of inertia and the radius of gyration of the shaded area with respect to the x and y axes.



SOLUTION



$$I_x = (\bar{I}_x + Ad_y^{\,2})_A + (\bar{I}_x + Ad_y^{\,2})_B + (\bar{I}_x + Ad_y^{\,2})_C$$

(In the B term, d_y is marked 0.)

$$= [\frac{1}{12}(24)(6)^3 + (24\times 6)(27)^2]_A$$

$$+ [\frac{1}{12}(8)(48)^3 + 0]_B$$

$$+ [\frac{1}{12}(48)(6)^3 + (48\times 6)(27)^2]_C$$

$$I_x = 390\text{x}10^3 \text{ mm}^4$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{390\times 10^3}{[(24\times 6)+(8\times 48)+(48\times 6)]}} = 21.9 \; mm$$

$$I_y = (\bar{I}_y + Ad_x^{\,2})_A + (\bar{I}_y + Ad_x^{\,2})_B + (\bar{I}_y + Ad_x^{\,2})_C$$

(In each term, d_x is marked 0.)

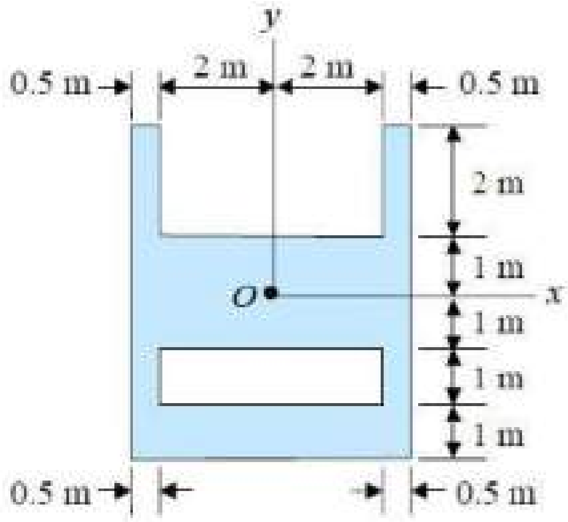
$$= [\frac{1}{12}(6)(24)^3]_A + [\frac{1}{12}(48)(8)^3]_B + [\frac{1}{12}(6)(48)^3]_C$$

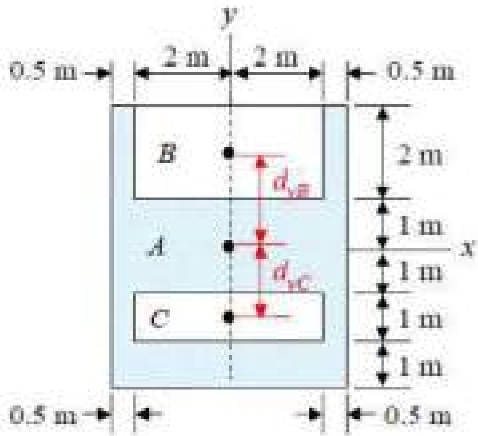
$$I_y = 64.3\text{x}10^3 \text{ mm}^4$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{64.3\times 10^3}{[(24\times 6)+(8\times 48)+(48\times 6)]}} = 8.87 \; mm$$

Example

Determine the moments of inertia and the radius of gyration of the shaded area with respect to the x and y axes.





$$I_x = (\bar{I}_x + Ad_y^2)_{A5\times6} - (\bar{I}_x + Ad_y^2)_{B4\times2} - (\bar{I}_x + Ad_y^2)_{C4\times1}$$

(with $d_y = 0$ for A)

$$= [\frac{1}{12}(5)(6)^3 + 0]_A - [\frac{1}{12}(4)(2)^3 + (2\times4)(2)^2]_B$$

$$-[\frac{1}{12}(4)(1)^3 + (4\times1)(1.5)^2]_C$$

$$I_x = 46 \text{ m}^4$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{46}{[(5\times6)-(4\times2)-(4\times1)]}} = 1.599\ m$$

$$I_y = (\bar{I}_y + Ad_x^2)_A - (\bar{I}_y + Ad_x^2)_B - (\bar{I}_y + Ad_x^2)_C$$

(with $d_x = 0$ for A, B and C)

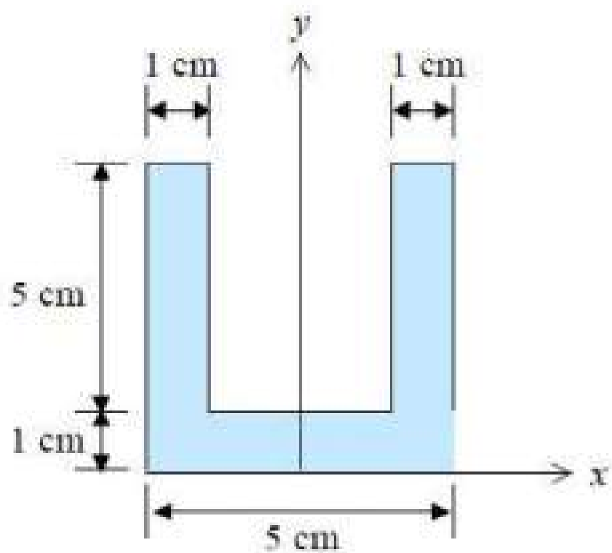
$$= [\frac{1}{12}(6)(5)^3]_A - [\frac{1}{12}(2)(4)^3]_B - [\frac{1}{12}(1)(4)^3]_C$$

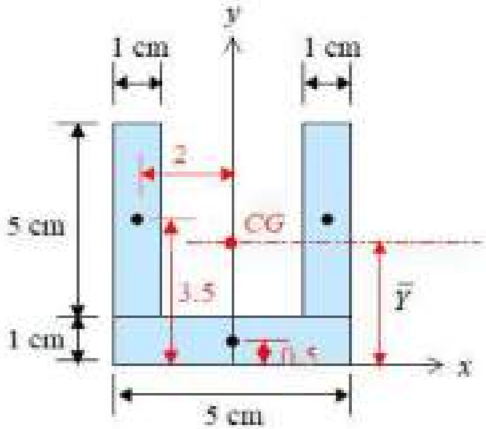
$$I_y = 46.5 \text{ m}^4$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{46.5}{[(5\times6)-(4\times2)-(4\times1)]}} = 1.607\ m$$

Example

Determine the moments of inertia and the radius of gyration of the shaded area with respect to the x and y axes and at the centroidal axes.





$$\bar{Y}\sum A = \sum \bar{y}A$$

$$\bar{Y} = \frac{2[(3.5)(5\times1)] + (0.5)(1\times5)}{3(5\times1)}$$
$$= 2.5\ cm$$

- **Moments of inertia about *x* axis**

$$I_x = 2[(\frac{1}{12}(1)(5)^3 + (5\times1)(3.5)^2] + \frac{1}{3}(5)(1)^3$$
$$= 145\ cm^4$$

- **Moments of inertia about centroid**

$$\bar{I}_x = I_x - Ad_y^{\,2}$$
$$= 145 - (15)(2.5)^2$$
$$= 51.25\ cm^4$$

OR

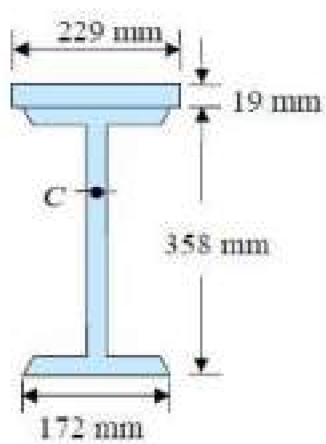
$$\bar{I}_x = 2[(\frac{1}{12}(1)(5)^3 + (5\times1)(1)^2]$$
$$+ [(\frac{1}{12}(5)(1)^3 + (5\times1)(2)^2]$$
$$= 51.25\ cm^4$$

$$\bar{I}_y = I_y = 2[(\frac{1}{12}(5)(1)^3 + (5\times1)(2)^2] + \frac{1}{12}(1)(5)^3$$
$$= 51.25\ cm^4$$

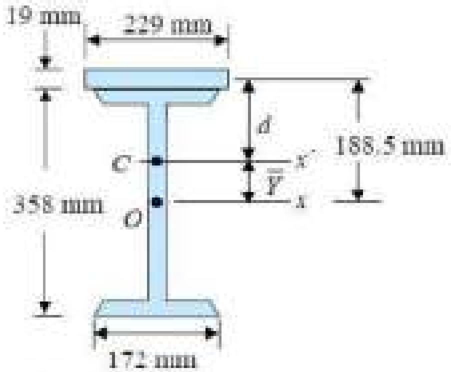
$$\bar{k}_x = \bar{k}_y = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{51.25}{15}} = 1.848\ cm$$

Example

The strength of a W360 x 57 rolled-steel beam is increased by attaching a 229 mm x 19 mm plate to its upper flange as shown. Determine the moment of inertia and the radius of gyration of the composite section with respect to an axis which is parallel to the plate and passes through the centroid *C* of the section.



SOLUTION



- **Centroid**

The wide-flange shape of *W*360 x 57 found by referring to Fig. 9.13

$A = 7230\ mm^2 \quad \bar{I}_x = 160.2\ mm^4$

$A_{plate} = (229)(19) = 4351\ mm^2$

$\bar{Y}\Sigma A = \Sigma \bar{y} A$

$\bar{Y}(4351 + 7230) = (188.5)(4351) + (0)(7230)$

$\bar{Y} = 70.8\ mm$

- **Moment of Inertia**

$$I_{x'} = (I_{x'})_{plate} + (I_{x'})_{wide-flange}$$

$$= (\bar{I}_{x'} + Ad^2)_{plate} + (\bar{I}_{x'} + A\bar{Y}^2)_{wide-flange}$$

$$= \left[\frac{1}{12}(229)(19)^3 + (4351)(188.5 - 70.8)^2\right] + \left[160.2 \times 10^6 + (7230)(70.8)^2\right]$$

$$= 256.8 \times 10^6\ mm^4$$

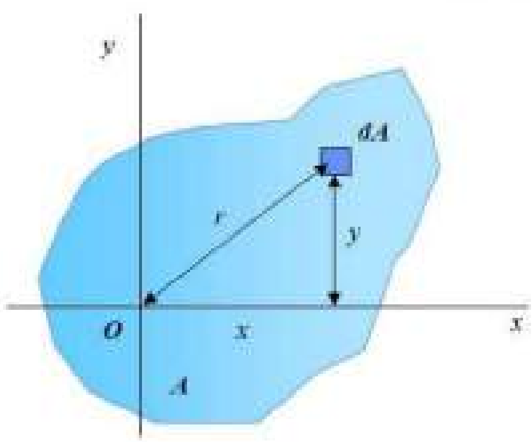
$$I_{x'} = 257 \times 10^6\ mm^4$$

- **Radius of Gyration**

$$k_{x'}^2 = \frac{I_{x'}}{A} = \frac{256.8 \times 10^6}{(4351 + 7230)}$$

$$k_{x'} = 149\ mm$$

Polar Moment of Inertia



The *polar moment of inertia of an area A* with respect to the pole O is defined as

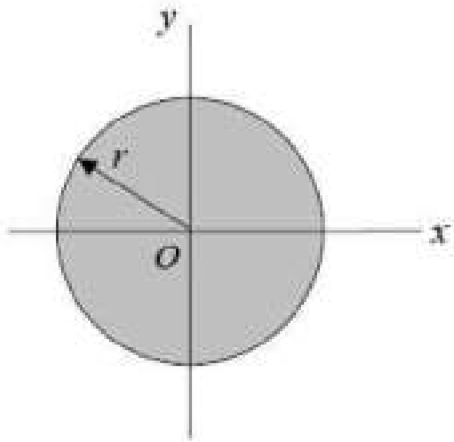
$$J_O = \int r^2\, dA$$

The distance from O to the element of area dA is r. Observing that $r^2 = x^2 + y^2$, we established the relation

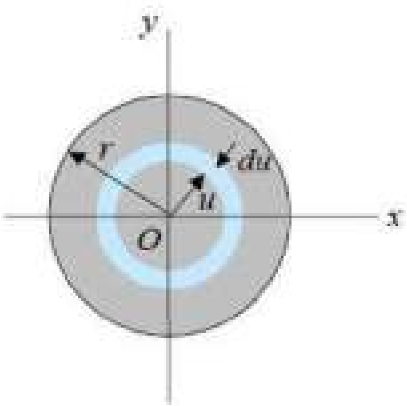
$$J_O = I_x + I_y$$

Example

(*a*) Determine the centroidal polar moment of inertia of a circular area by direct integration. (*b*) Using the result of part *a*, determine the moment of inertia of a circular area with respect to a diameter.



SOLUTION



a. Polar Moment of Inertia.

$$dJ_O = u^2\,dA \qquad dA = 2\pi u\,du$$

$$J_O = \int dJ_O = \int_0^r u^2(2\pi u\,du) = 2\pi\int_0^r u^3\,du$$

$$J_O = \frac{\pi}{2}r^4$$

b. Moment of Inertia with Respect to a Diameter.

$$J_O = I_x + I_y = 2I_x$$

$$\frac{\pi}{2}r^4 = 2I_x$$

$$I_{diameter} = I_x = \frac{\pi}{4}r^4$$

Centroids and Moments of Inertia of Engineering Sections

CENTROID OF PLANE FIGURES

4.1 Centre of Gravity:

Everybody is attracted towards the centre of the earth due gravity. The force of attraction is proportional to mass of the body. Everybody consists of innumerable particles, however the entire weight of a body is assumed to act through a single point and such a single point is called centre of gravity.

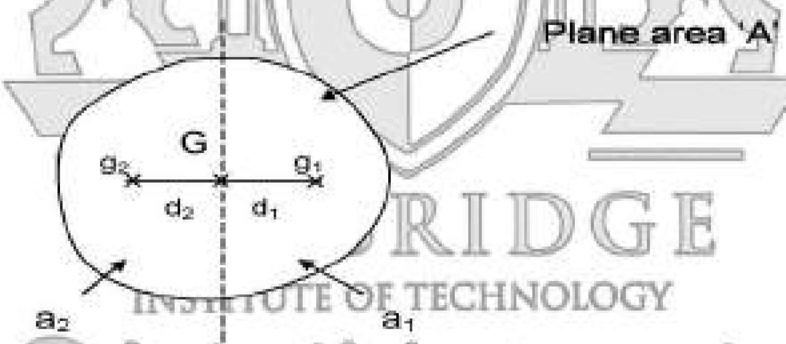
Every body has one and only centre of gravity.

4.2 Centroid:

In case of plane areas (bodies with negligible thickness) such as a triangle quadrilateral, circle etc., the total area is assumed to be concentrated at a single point and such a single point is called centroid of the plane area.

The term centre of gravity and centroid has the same meaning but the following differences.

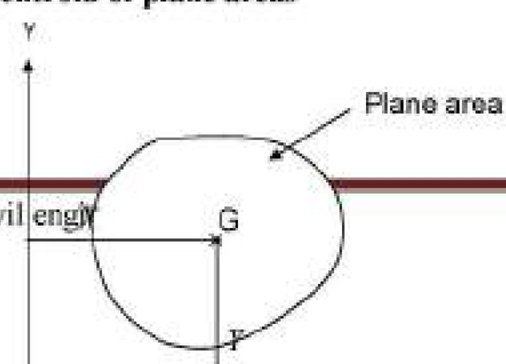
1. Centre of gravity refer to bodies with mass and weight whereas, centroid refers to plane areas.
2. centre of gravity is a point is a point in a body through which the weight acts vertically downwards irrespective of the position, whereas the centroid is a point in a plane area such that the moment of areas about an axis through the centroid is zero



Note: In the discussion on centroid, the area of any plane figure is assumed as a force equivalent to the centroid referring to the above figure G is said to be the centroid of the plane area A as long as

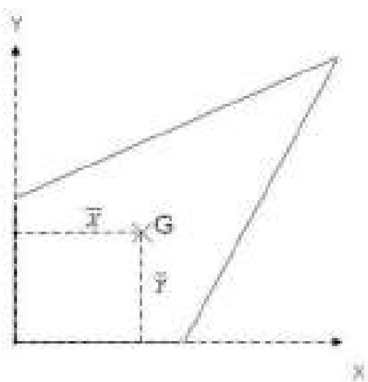
$a_1 d_1 - a_2 d_2 = 0.$

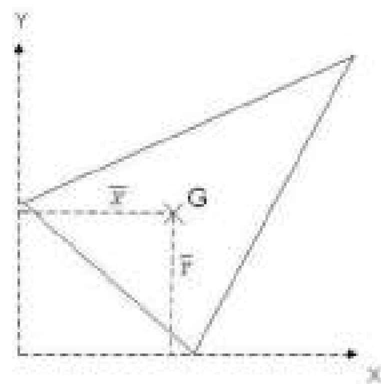
4.3 Location of centroid of plane areas



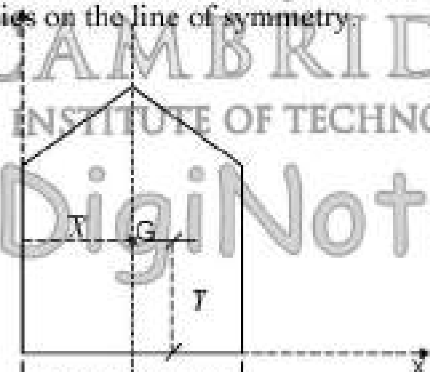
The position of centroid of a plane area should be specified or calculated with respect to some reference axis i.e. X and Y axis. The distance of centroid G from vertical reference axis or Y axis is denoted as X and the distance of centroid G from a horizontal reference axis or X axis is denoted as Y.

While locating the centroid of plane areas, a bottommost horizontal line or a horizontal line through the bottommost point can be made as the X – axis and a leftmost vertical line or a vertical line passing through the leftmost point can be made as Y- axis.





In some cases the given figure is symmetrical about a horizontal or vertical line such that the centroid of the plane area lies on the line of symmetry.



The above figure is symmetrical about a vertical line such that G lies on the line of symmetry. Thus

X= b/2.

Y=?

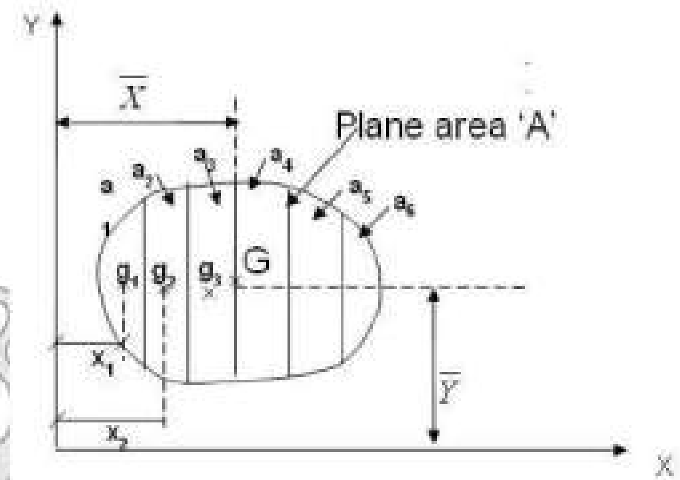
The centroid of plane geometric area can be located by one of the following methods

a) Graphical methods

b) Geometric consideration
c) Method of moments

The centroid of simple elementary areas can be located by geometric consideration. The centroid of a triangle is a point, where the three medians intersect. The centroid of a square is a point where the two diagonals bisect each other. The centroid of a circle is centre of the circle itself.\

METHOD OF MOMENTS TO LOCATE THE CENTROID OF PLANE AREAS



Let us consider a plane area A lying in the XY plane. Let G be the centroid of the plane area. It is required to locate the position of centroid G with respect to the reference axis like Y- axis and Xi- axis i.e, to calculate X and Y. Let us divide the given area A into smaller elemental areas a_1, a_2, a_3 …….. as shown in figure. Let g_1 ,g_2, g_3…… be the centroids of elemental areas a_1, a_2, a_3 …….. etc.

Let x_1, x_2, x_3 etc be the distance of the centroids g_1 g_2 g_3 etc. from Y- axis is $=A.\overline{X}$ -----(1)

The sum of the moments of the elemental areas about Y axis is

$a_1 . x_1 + a_2 . x_2 + a_3 . x_3 +$ ……………….(2)

Equating (1) and (2)

$A . \overline{X} = a_1 . x_1 + a_2 . x_2 + a_3 . x_3 +$ ………..

$$\overline{X} = \frac{a_1 . x_1 + a_2 . x_2 + a_3 . x_3 + \ldots\ldots\ldots}{A}$$

$$\overline{X} = \frac{\sum (ax)}{A} \quad \text{or} \quad \overline{X} = \frac{\int x dA}{A}$$

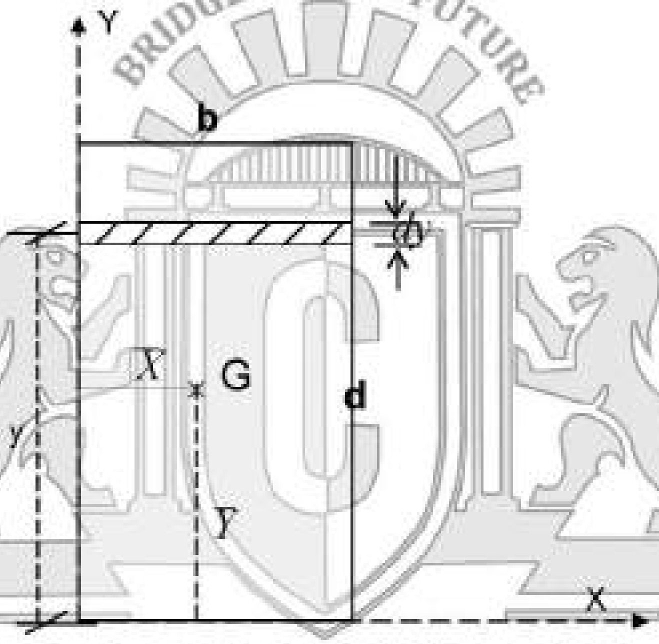
Where a or dA represents an elemental area in the area A , x is the distance of elemental area from Y axis.

Similarly

$$\bar{Y} = \frac{a1.y1+a2.y2+a3.y3+........}{A}$$

$$\bar{Y} = \frac{\sum(a.y)}{A} \text{ or } \bar{Y} = \frac{\int y dA}{A}$$

TO LOCATE THE CENTROID OF A RECTANGLE FROM THE FIRST PRINCIPLE (METHOD OF MOMENTS)



Let us consider a rectangle of breadth b and depth d. let g be the centroid of the rectangle. Let us consider the X and Y axis as shown in the figure.
Let us consider an elemental area dA of breadth b and depth dy lying at a distance of y from the X axis.

W.K.T

$$\overline{Y} = \frac{\int_0^d y.dA}{A}$$

A = b . d
dA = b . dy

$$\overline{Y} = \frac{\int_0^d y.(b.dy)}{b.d}$$

$$\Rightarrow$$

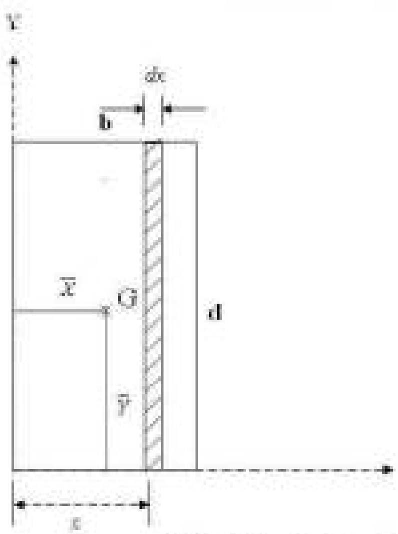
$$\overline{Y} = \frac{1}{d}\int_0^d y.dy$$

$$\overline{Y} = \frac{1}{d}\left[\frac{y^2}{2}\right]_0^d$$

$$\overline{Y} = \frac{1}{d}\left[\frac{d^2}{2}\right]$$

$$\overline{Y} = \frac{d}{2}$$

Similarly



$$\overline{X} = \frac{\int_0^b x.dA}{A}$$

A = b . d
dA = dx . d

$$\overline{X} = \frac{\int_0^b x.(dx.d)}{b.d}$$

$$\Rightarrow$$

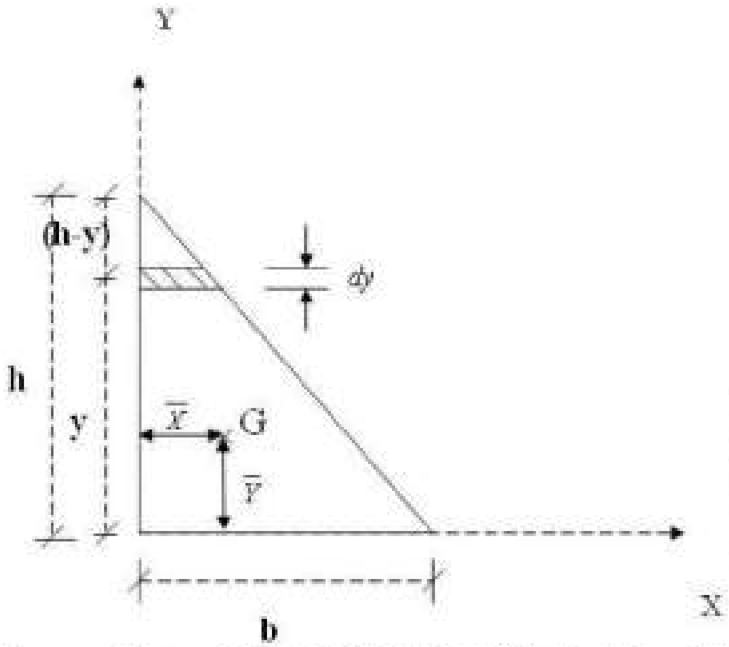
$$\overline{X} = \frac{1}{d}.\int_0^b x.dx$$

$$\overline{X} = \frac{1}{b}.\left[\frac{x^2}{2}\right]_0^b$$

$$\overline{X} = \frac{1}{b}.\left[\frac{b^2}{2}\right]$$

$$\overline{X} = \frac{b}{2}$$

Centroid of a triangle



Let us consider a right angled triangle with a base b and height h as shown in figure. Let G be the centroid of the triangle. Let us consider the X- axis and Y- axis as shown in figure.
Let us consider an elemental area dA of width b1 and thickness dy, lying at a distance y from X-axis.

W.K.T

$$\overline{Y} = \frac{\int_0^h y \, dA}{A}$$

$$A = \frac{bh}{2}$$

$$dA = b_1 \cdot dy$$

$$\overline{Y} = \frac{\int_0^h y.(b_1 . dy)}{\frac{bh}{2}}$$ [as x varies b1 also varies]

$$\bar{Y}=\frac{2}{h}\int_0^h\left(y-\frac{y^2}{h}\right)dy$$

$$\bar{Y}=\frac{2}{h}\left[\frac{y^2}{2}-\frac{y^3}{3h}\right]_0^h$$

$$\bar{Y}=\frac{2}{h}\left[\frac{h^2}{2}-\frac{h^3}{3h}\right]$$

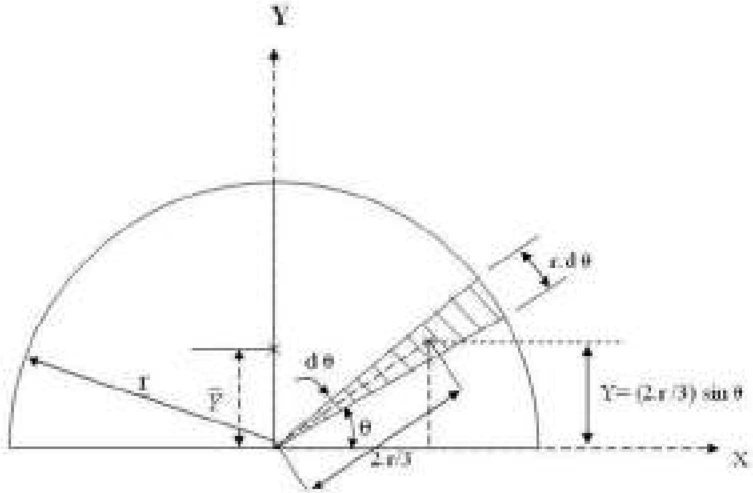
$$\bar{Y}=\frac{2}{h}\left[\frac{h^2}{2}-\frac{h^2}{3}\right]$$

$$\bar{Y}=2h\left[\frac{1}{2}-\frac{1}{3}\right]$$

$$\bar{Y}=\frac{2h}{6}$$

$\bar{Y}=\frac{h}{3}$ similarly $\bar{X}=\frac{b}{3}$

Centroid of a semi circle



$\overline{X} = 0$

Let us consider a semi-circle, with a radius 'r'.

Let 'O' be the centre of the semi-circle .let 'G' be centroid of the semi-circle. Let us consider the x and y axes as shown in figure.

Let us consider an elemental area 'dA' with centroid 'g' as shown in fig. Neglecting the curvature, the elemental area becomes an isosceles triangle with base r.dθ and height 'r'.

Let y be the distance of centroid 'g' from x axis.

Here $y = \frac{2r}{3}.\sin\theta$

W K T

$$\overline{Y} = \frac{\int y\,dA}{A}$$

$$A = \frac{\pi r^2}{2}$$

$$\overline{Y} = \frac{\int y\,dA}{A}$$

Centroid of a quarter circle

$$\overline{Y} = \frac{\int \frac{2r}{3}.\sin\theta\,dA}{A}$$

$$dA = \frac{1}{2}.r.d\theta.r$$

$$dA = \frac{r^2}{2}d\theta$$

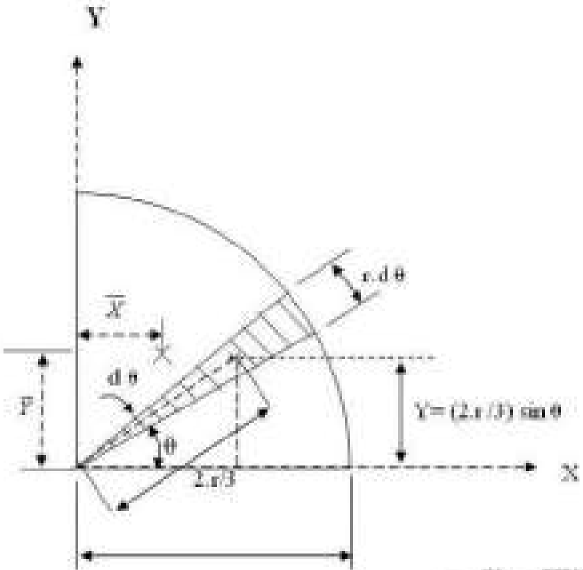
$$= \frac{2}{3\pi}\int r\sin\theta\,d\theta$$

$$= \frac{2r}{3\pi}\int_0^{\pi}\sin\theta\,d\theta$$

$$= \frac{2r}{3\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{2r}{3\pi}[1+1]$$

$$\overline{Y} = \frac{4r}{3\pi}$$



Let us consider a quarter circle with radius r. Let 'O' be the centre and 'G' be the centroid of the quarter circle. Let us consider the x and y axes as shown in figure.
Let us consider an elemental area 'dA' with centroid 'g' as shown in fig.
Let 'y' be the distance of centroid 'g' from x axis. Neglecting the curvature, the elemental area becomes an isosceles triangle with base r.dθ and height 'r'.

Here $y = \frac{2r}{3}.\sin\theta$

WKT

$$\overline{Y} = \frac{\int y dA}{A}$$

$$A = \frac{\pi r^2}{2}$$

$$\overline{Y} = \frac{\int y dA}{A}$$

$$\overline{Y} = \frac{\int \frac{2r}{3}.\sin\theta.dA}{A}$$

$$dA = \frac{1}{2} r d\theta r$$

$$dA = \frac{r^2}{2}.d\theta$$

$$\overline{Y} = \frac{\int \frac{2r}{3}.\sin\theta.\frac{r^2}{2}.d\theta}{\frac{\pi r^2}{4}}$$

$$\frac{4r}{3\pi}\int_0^{\pi/2} \sin\theta.d\theta$$

$$= \frac{2r}{3\pi}\left[-\cos\theta\right]_0^{\pi/2}$$

$$= \frac{4r}{3\pi}[0+1]$$

$$\overline{Y} = \frac{4r}{3\pi}$$

Similarly

$$\overline{X} = \frac{4r}{3\pi}$$

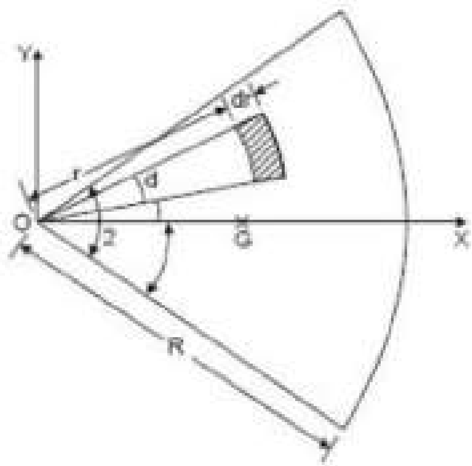
Centroid of Sector of a Circle

Consider the sector of a circle of angle 2α as shown in Fig. Due to symmetry, centroid lies on x axis. To find its distance from the centre O, consider the elemental area shown.

Area of the element $= r d\theta\, dr$

Its moment about y axis

$$= r d\theta \times dr \times r \cos\theta$$
$$= r^2 \cos\theta\, dr d\theta$$



$\therefore$ Total moment of area about y axis

$$= \int_{-\alpha}^{\alpha}\int_{0}^{R} r^2 \cos\theta\, dr d\theta$$

$$= \left[\frac{r^3}{3}\right]_0^R \left[\sin\theta\right]_{-\alpha}^{\alpha}$$

$$= \frac{R^3}{3}\, 2\sin\alpha$$

Total area of the sector

$$= \int_{-\alpha}^{\alpha}\int_{0}^{R} r\, dr d\theta$$

$$= \int_{-\alpha}^{\alpha}\left[\frac{r^2}{2}\right]_0^R d\theta$$

$$= \frac{R^2}{2}\left[\theta\right]_{-\alpha}^{\alpha}$$

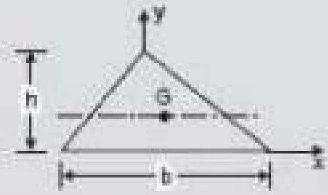
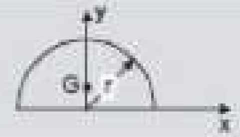
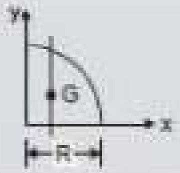
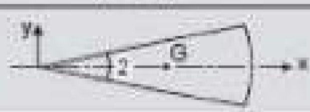
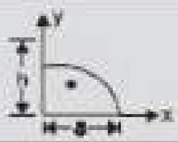
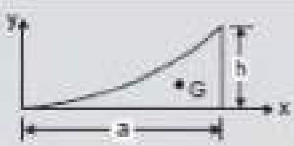
$$= R^2\alpha$$

$\therefore$ The distance of centroid from centre O

$$= \frac{\text{Moment of area about } y \text{ axis}}{\text{Area of the figure}}$$

$$= \frac{\frac{2R^3}{3}\sin\alpha}{R^2\alpha} = \frac{2R}{3\alpha}\sin\alpha$$

Centroid of Some Common Figures

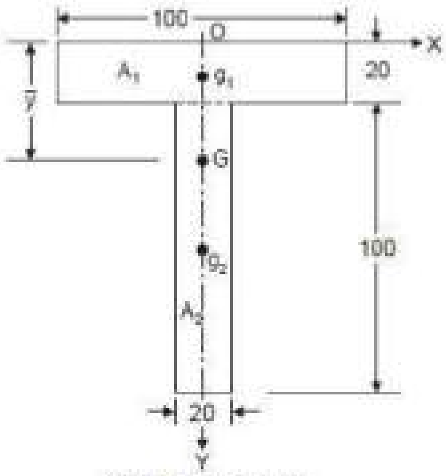
Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Triangle		—	$\frac{h}{3}$	$\frac{bh}{2}$
Semicircle		0	$\frac{4R}{3\pi}$	$\frac{\pi R^2}{2}$
Quarter circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$\frac{\pi R^2}{4}$
Sector of a circle		$\frac{2R}{3\alpha}\sin\alpha$	0	αR^2
Parabola		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Semi parabola		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$

4.5 Centroid of Composite Sections

In engineering practice, use of sections which are built up of many simple sections is very common. Such sections may be called as built-up sections or composite sections. To locate the centroid of composite sections, one need not go for the first principle (method of integration). The given composite section can be split into suitable simple figures and then the centroid of each simple figure can be found by inspection or using the standard formulae listed in the table above. Assuming the area of the simple figure as concentrated at its centroid, its moment about an axis can be found by multiplying the area with distance of its centroid from the reference axis. After determining moment of each area about reference axis, the distance of centroid from the axis is obtained by dividing total moment of area by total area of the composite section.

PROBLEMS:

Q) Locate the centroid of the T-section shown in fig.



All dimensions in mm

Solution. Selecting the axis as shown in Fig. we can say due to symmetry centroid lies on y axis, *i.e.* $\bar{x} = 0$. Now the given T-section may be divided into two rectangles A_1 and A_2 each of size 100 × 20 and 20 × 100. The centroid of A_1 and A_2 are $g_1(0, 10)$ and $g_2(0, 70)$ respectively.

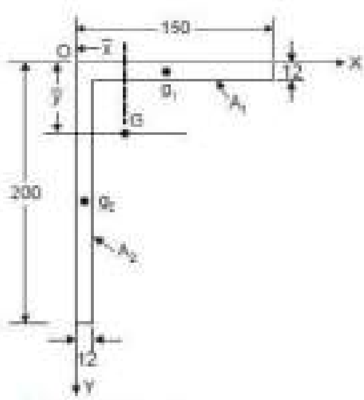
∴ The distance of centroid from top is given by:

$$\bar{y} = \frac{100 \times 20 \times 10 + 20 \times 100 \times 70}{100 \times 20 + 20 \times 100}$$

$$= 40 \text{ mm}$$

Hence, *centroid of T-section is on the symmetric axis at a distance 40 mm from the top.* **Ans.**

Q) Find the centroid of the unequal angle 200 × 150 × 12 mm, shown in Fig.



All dimensions in mm

Solution. The given composite figure can be divided into two rectangles:

$$A_1 = 150 \times 12 = 1800 \text{ mm}^2$$

$$A_2 = (200 - 12) \times 12 = 2256 \text{ mm}^2$$

Total area $A = A_1 + A_2 = 4056 \text{ mm}^2$

Selecting the reference axis x and y as shown in Fig. 2.30. The centroid of A_1 is g_1 (75, 6) and that of A_2 is:

$$g_2\left[6, 12 + \frac{1}{2}(200 - 12)\right]$$

i.e., g_2 (6, 106)

$$\therefore \quad \bar{x} = \frac{\text{Movement about } y \text{ axis}}{\text{Total area}}$$

$$= \frac{A_1x_1 + A_2x_2}{A}$$

$$= \frac{1800 \times 75 + 2256 \times 6}{4056} = 36.62 \text{ mm}$$

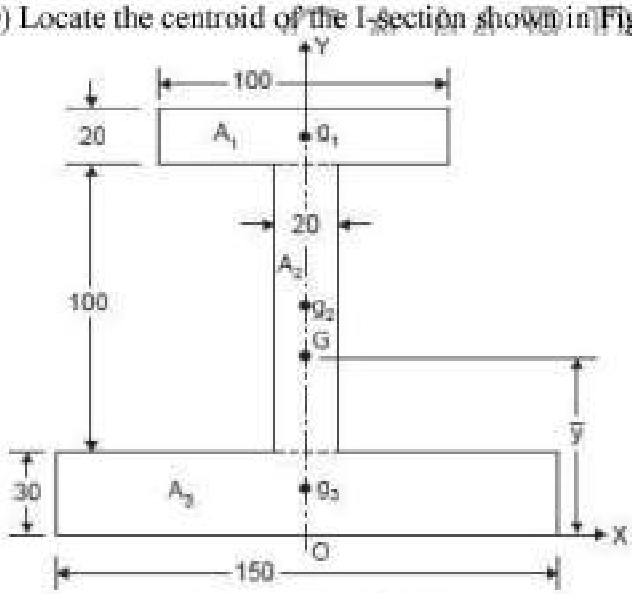
$$\bar{y} = \frac{\text{Movement about } x \text{ axis}}{\text{Total area}}$$

$$= \frac{A_1y_1 + A_2y_2}{A}$$

$$= \frac{1800 \times 6 + 2256 \times 106}{4056} = 61.62 \text{ mm}$$

Thus, the centroid is at $\bar{x}$ = 36.62 mm and $\bar{y}$ = 61.62 mm as shown in the figure **Ans.**

Q) Locate the centroid of the I-section shown in Fig.



All dimensions in mm

Solution. Selecting the co-ordinate system as shown in Fig. due to symmetry centroid must lie on y axis,

i.e., $\bar{x} = 0$

Now, the composite section may be split into three rectangles

$$A_1 = 100 \times 20 = 2000 \text{ mm}^2$$

Centroid of A_1 from the origin is:

$$y_1 = 30 + 100 + \frac{20}{2} = 140 \text{ mm}$$

Similarly $A_2 = 100 \times 20 = 2000 \text{ mm}^2$

$$y_2 = 30 + \frac{100}{2} = 80 \text{ mm}$$

$$A_3 = 150 \times 30 = 4500 \text{ mm}^2, \text{ and}$$

$$y_3 = \frac{30}{2} = 15 \text{ mm}$$

$\therefore$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A}$$

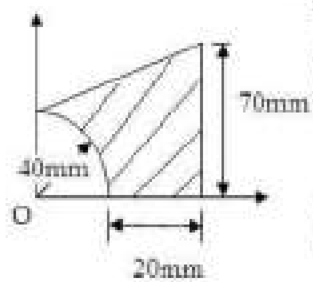
$$= \frac{2000 + 140 + 2000 \times 80 + 4500 \times 15}{2000 + 2000 + 4500}$$

$$= 59.71 \text{ mm}$$

Thus, *the centroid is on the symmetric axis at a distance 59.71 mm from the bottom* as shown in Fig. **Ans.**

1.Determine the centroid of the lamina shown in fig. wrt O.

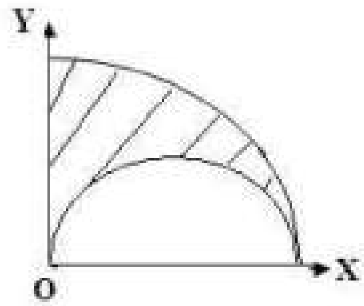
(June/July2009, June/July2013)



Component	Area (mm²)	X (mm)	Y (mm)	aX	aY
Quarter circle	-1256.64	16.97	16.97	-21325.2	-21325.2
Triangle	900	40	50	36000	45000
Rectangle	2400	30	20	72000	48000
	∑a= 2043.36			∑ aX = 86674.82	∑ aY = 71674.82

$\bar{X}$ = 42.42 mm; $\bar{Y}$ = 35.08 mm

Find the centroid of the shaded area shown in fig, obtained by cutting a semicircle of diameter 100mm from the quadrant of a circle of radius 100mm. (Jan 2011)



Component	Area (mm^2)	X (mm)	Y (mm)	aX	aY
Quarter circle	7853.98	42.44	42.44	333322.9	333322.9
Semi circle	-3926.99	50	21.22	-196350	-83330.7
	$\sum a$= 3926.99			$\sum$ aX = 136973.4	$\sum$ aY = 249992.2

X = 34.88 mm; Y = 63.66 mm

MODULE 5

KINEMATICS

INTRODUCTION TO DYNAMICS

Dynamics is the branch of science which deals with the study of behaviour of body or particle in the state of motion under the action of force system. The first significant contribution to dynamics was made by Galileo in 1564. Later, Newton formulated the fundamental laws of motion.

Dynamics branches into two streams called kinematics and kinetics.

Kinematics is the study of relationship between displacement, velocity, acceleration and time of the given motion without considering the forces that causes the motion, or Kinematics is the branch of dynamics which deals with the study of properties of motion of the body or particle under the system of forces without considering the effect of forces.

Kinetics is the study of the relationships between the forces acting on the body, the mass of the body and the motion of body, or Kinetics is the branch of dynamics which deals with the study of properties of motion of the body or particle in such way that the forces which cause the motion of body are mainly taken into consideration.

TECHNICAL TERMS RELATED TO MOTION

Motion: A body is said to be in motion if it is changing its position with respect to a reference point.

Path: It is the imaginary line connecting the position of a body or particle that has been occupied at different instances over a period of time. This path traced by a body or particle can be a straight line/liner or curvilinear.

Displacement and Distance Travelled

P —> Position of the particle at any time t1

x_1—> Displacement of particle measured in +ve direction of O

Displacement is a vector

In this case the total distance travelled by a particle from point O to P to P_1 and back to O is

quantity, measure of the interval between two locations or two points, measured along the shortest path connecting them. Displacement can be positive or negative.

Distance is a scalar quantity, measure of the interval between two locations measured along the actual path connecting them. Distance is an absolute quantity and always positive.

Average velocity: When an object undergoes change in velocities at different instances, the average velocity is given by the sum of the velocities at different instances divided by the number of instances. That is, if an object has different velocities v_1, v_2, v_3, ... ,v_n, at times t =

$$V = (v_1+v_2+v_3+....v_n)/n$$

A particle in a rectilinear motion occupies a certain position on the straight line. To define this position P of the particle we have to choose some convenient reference point O called origin (Figure 5.1). The distance x_1 of the particle from the origin is called displacement.

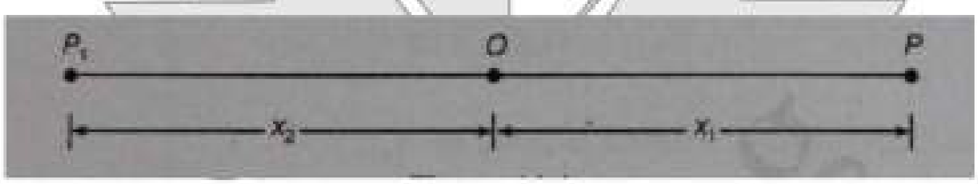


Figure 5.1

Let,

Total distance travelled = $x_1 + x_1 + x_2 + x_2 = 2(x_1 + x_2$ Whereas the net displacement is zero

Rectilinear Motion notes:

"Rectilinear motion" represents an object moving back and forth linearly (either up/down or left/right) but not both at the same time (we will deal with that when we do vectors in BC).

Let s(t) represent the position of an object at any time t. Position can be positive or negative – indicating whether above (rt) or below (lt) of the origin.

Average velocity on an interval [a, b], would be the change in position divided by the change in

Velocity: Rate of change of **displacement with respect to time is called velocity denoted by** v. Mathematically v = dx/dt

Average velocity: When an object **undergoes change in velocities at different instances,** the average velocity is given by the sum of the velocities at different instan**ces divided by the** number of instances. That is, if an object has different velocities v_1, v_2, v_3, … ,v_n, at times t =

$$V = (v_1+v_2+v_3+\dots v_n)/n$$

Instantaneous velocity: It is the velocity of moving **particle at a certain instant of time. To**

Instantaneous velocity $v = \Delta t \; 0 \; \Delta x/\Delta t$

Speed: Rate of change of distance travelled by the particle with respect to time is called

Acceleration: Rate of change of velocity with respect to time is called acceleration Mathematically a = dv/dt

Average Acceleration

Consider a particle P situated at a distances of x from O at any instant of time t having a velocity v. Let P_1 **be the new position of particle at a distance of (x + Δx) from origin with a** (v + Δv). See Figure 5.2.

Figure 5.2

time: $\frac{s(b)-s(a)}{b-a}$.

v(t) = velocity of the object at any time t, or the instantaneous rate of change of position with **respect to time = derivative of position (s'(t)). This also can be positive or negative depending** upon the direction of travel: + up/right, - down/left.

Speed = absolute value of velocity

a(t) = acceleration of the object at any time t, or the instantaneous rate of change of velocity = **derivative of velocity, v'(t) = second derivative of position, s"(t). Acceleration can also be** positive or negative depending on the rate of change of velocity.

When discussing what is **happening with an object, usually helpful to know if it is "speeding up" or "slowing down" = speed** increasing or decreasing. To do this, must look at signs of BOTH velocity and acceleration and compare. If same signs (ie. Both positive or both negative) then object is speeding up (speed is increasing), if opposite signs (one positive and the other negative) then the object is slowing down (speed is decreasing).

To put all of this together and "describe the motion" we like to make a chart. See examples:

Ex. The position function s(t) of a point P is given by $s(t) = t^3 - 12t^2 + 36t - 20$, with t in seconds and s(t) in centimeters. Describe the motion during the interval [-1, 9].

***Note: if they don't give an interval, do $(-\infty,\infty)$**

****Find v(t) and a(t) and do number lines!!!**

$v(t) = 3t^2 - 24t + 36 = 3(t-2)(t-6) = 0 \Rightarrow t = 2, t = 6$ -1 + 2 - 6 + 9

$a(t) = 6t - 24 = 0 \Rightarrow t = 4$ -1 - 4 + 9

****I like to line up my # lines****

****Make a chart – break up your intervals anywhere v(t) = 0 or a(t) = 0. ****

t	v(t)	Direction	a(t)	Speed inc/dec
(-1,2)	+	Right/up	-	Dec
t=2	0	changing	-	0
(2,4)	-	Left/down	-	Increasing
t=4	-	Left/down	0	Constant
(4,6)	-	Left/down	+	Decreasing
t=6	0	Changing	+	0
(6,9)	+	Right/up	+	Increasing

Note that when signs of v(t) and a(t) are the same, the speed is increasing. When the signs are the opposite, the speed is decreasing. Remember that speed = absolute value of v(t), so if v(t) =0 then speed=0 too. When a(t) = 0, the object is not accelerating so it is at a **constant speed.

This chart "describes the motion" – this is all you need to provide. You should show all derivatives and number lines.

Ex. Suppose a weight is oscillating on a spring and $s(t) = 10\cos(\frac{\pi}{6}t)$ where t is in seconds and s(t) is in centimeters. Describe the motion on the interval [-1, 13].

$v(t) = -\frac{5\pi}{3}\sin(\frac{\pi}{6}t) = 0$ => t = 0, 6, 12 -1 + 0 - 6 + 12 - 13

$a(t) = -\frac{5\pi^2}{18}\cos(\frac{\pi}{6}t) = 0$ => t = 3, 9 -1 - 3 + 9 - 13

t	v(t)	Direction	a(t)	Speed inc/dec
(-1,0)	+	Up	-	Dec
t = 0	0	Changing	-	0
(0, 3)	-	Down	-	Inc
t=3	-	Down	0	Constant
(3,6)	-	Down	+	Dec
t=6	0	Changing	+	0
(6,9)	+	Up	+	Inc
t=9	+	Up	0	Constant
(9,12)	+	Up	-	Dec
t=12	0	Changing	-	0
(12,13)	-	down	-	Inc

Should start to notice that charts USUALLY follow a pattern – which should make sense if you picture an object moving back and forth along a line

Ex. A projectile is fired straight upward from a 50 ft tall building with a velocity of 100 ft/sec. From physics, its distance above the ground at any time after t seconds is given by $s(t) = -16t^2 + 100t + 50$.

a) Find the time and velocity at which the projectile hits the ground.
b) Find the maximum altitude (height) achieved by the projectile.
c) Find the acceleration at any time t.

a) Need to figure out when hits ground => s(t) = 0 => Use quadratic formula t = 6.7153517 sec. Then find v(t) – take the derivative – and plug in 6.7153517 sec.
v(t) = -32t + 100 => v(6.7153517) = - _________ ft/s

b) Max height occurs when projectile is changing direction - when v(t) = 0. So find this and plug the time back into s(t) to get the height.

v(t) = -32t + 100 = 0 => t = 25/8 sec. => s(25/8) = ________ ft

c) **a(t) = v'(t) = s''(t) =** -32 ft/s^2 . This should make sense to anyone who took physics as this is the gravitational constant when in these units. If were in meters, then it would have been -9.8 m/s^2.

** If looking at the graph of *v*(t), could determine info about s(t) just like determined **information about f(x) given f '(x) – it's the same thing! We will look at some of these in class** next week as a review.